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AN ITERATIVE SOLUTION OF A VARIATIONAL INEQUALITY FOR CERTAIN MONOTONE OPERATORS IN HILBERT SPACE

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Let A be a multivalued monotone operator on a real Hilbert space H and let C be a nonempty closed convex subset of $D(A)$. If $f \in H$, by a solution of the variational inequality

$$(1) \qquad (z_0 - f, x - u_0) \geqslant 0 \quad \forall x \in C,$$

we mean a pair (or, sometimes, just the first component of a pair) $[u_0, z_0] \in A$ satisfying (1) such that $u_0 \in C$. We denote the set of solutions u_0 by E. We shall assume the existence of a solution of (1) and show how to construct it as the weak limit of a sequence $\{x_n\}$ satisfying

$$(2) \qquad x_{n+1} = P(x_n - t_n(v_n - f)), \qquad v_n \in Ax_n,$$

where $\{t_n\} \subset [0, \infty)$ and P is the proximity mapping of H onto C. For conditions sufficient to guarantee $E \neq \varnothing$, see Browder **[4]**, Lions **[10]**.

THEOREM 1. *Suppose there exists $u_0 \in E$ such that*

$$(3) \qquad \{(v - f, x - u_0) = 0, x \in C, v \in Ax\} \Rightarrow x \in E.$$

If, in (2), $\Sigma t_n = \infty$, $\Sigma \|t_n(v_n - f)\|^2 < \infty$, and $\{v_n\}$ is bounded, then $\{x_n\}$ converges weakly to a point of E.

Note, in particular, that if A is bounded on C, then for any nonnegative sequence $\{t_n\}$ in $l^2 \setminus l^1$ the conditions on $\{t_n\}$ and $\{v_n\}$ are automatically satisfied.

THEOREM 2. *If A has the property*

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