balanced, and a more widely appealing book, and it is a shame that the opportunity has been missed.

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Monotone matrix functions and analytic continuation, by W. F. Donoghue, Jr., Springer-Verlag, New York, Heidelberg, Berlin, 1974, 182 pp., $19.70

In a 1934 article Charles Loewner posed and solved the following problem: Characterize the class $P_n(a, b)$ of real-valued functions on the interval (a, b) that are *monotone matrix functions of order n*. This means that whenever A, B are n-by-n Hermitian matrices with spectrum in (a, b) and $A \geqq B$ (i.e. $A-B$ is positive definite), then $f(A) \geqq f(B)$. As usual, $f(A)$ is defined as the Hermitian matrix whose eigenvectors are the same as those of A and whose eigenvalues are gotten from those of A by applying f. Loewner showed that for $n \geqq 2$ such a function is automatically continuously differentiable and, regarded as a function from the linear space of n-by-n Hermitian matrices to itself, its derivative at $A = \operatorname{diag}(\lambda_1, \cdots, \lambda_n)$ sends the matrix (X_{jk}) to the matrix $([\lambda_j, \lambda_k]_f X_{jk})$, where

$$[x, y]_f = \begin{cases} \dfrac{f(x)-f(y)}{x-y} & \text{if } x \neq y, \\ f'(x) & \text{if } x = y. \end{cases}$$

So a necessary and sufficient condition for monotonicity of order n is the positive definiteness of the matrix $[\xi_j, \xi_k]_f$ for every choice of $\xi_1, \cdots, \xi_n \in (a, b)$. An equivalent condition is the positive definiteness of $[\xi_j, \eta_k]_f$ for every $a < \xi_1 < \eta_1 < \xi_2 < \cdots < \eta_n < b$; in fact Loewner starts with proving the necessity