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THE SELBERG TRACE FORMULA FOR CONGRUENCE SUBGROUPS

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1. **Introduction.** The Selberg trace formula for $SL(2, \mathbf{R})$ is commonly understood to be a non-Abelian analog of the Poisson summation formula. The formula arises from letting a Fuchsian group Γ act on the upper half-plane H and contains four basic contributions: identity, hyperbolic, elliptic, and parabolic [2, pp. 95–108], [3, pp. 72–79]. Because of its possible number-theoretic applications, it seems only natural to calculate the trace formula explicitly for various congruence subgroups of $SL(2, \mathbf{Z})$ and see what happens.

From the general theory, one knows that the parabolic (or arithmetic) contribution will be $\text{Tr}(M)$, where

$$M = \frac{1}{4\pi} \int_{-\infty}^{\infty} h(r) \Phi'(s) \Phi(s)^{-1} dr + \frac{1}{4} [I - \Phi(\frac{1}{2})] h(0) \\ - \left[g(0) \ln 2 + \frac{1}{2\pi} \int_{-\infty}^{\infty} h(r) \frac{\Gamma'}{\Gamma} (1 + ir) dr \right] I.$$