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C^*-ALGEBRAS GENERATED BY COMMUTING ISOMETRIES. I[1]

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We announce results on structure and Fredholm theory for the C^*-algebra $A(V, W)$ generated by two isometries V and W on a separable Hilbert space with the properties (where $[A, B] \equiv AB - BA$) that $[V, W] = 0$ and $[V, W^*]$ is compact. Such algebras arise naturally in several contexts: first, in the stuuy of singular integral operators with generally discontinuous symbols; secondly, in the study of difference operators on certain domains in R^2. Proofs of the results described here and extensions to the case of more than two generators will appear elsewhere [1].

Let $L^2(T)$ denote the space of Lebesgue square-integrable complex-valued functions on the circle T and let $H^2(T)$ be the Hardy subspace of $L^2(T)$ with P_+ the orthogonal projection from $L^2(T)$ onto $H^2(T)$. Now for Φ in $L^\infty(T)$, we define the Toeplitz operator T_Φ by $T_\Phi f = P_+ \Phi f$. We recall than an inner function is an element of $H^2(T)$ which has modulus one a.e. The only continuous inner functions are the finite Blaschke products and it is a property of inner functions that their discontinuities are of the rapidly oscillating type rather than jumps.

Our analysis is based on a careful examination of the product of generators, VW. There is a reducing space $H = \{x: x = (VW)^n y_n \forall n > 0\}$ for VW so that $VW|_H$ is unitary and since $[V, W] = 0$, H reduces V and W with $V|_H$, $W|_H$ both unitary. Moreover, there is a unitary equivalence between $H^\perp$ and $H^2(T) \otimes l_2(S)$ where S is a subset of the integers Z so that $VW|_{H^\perp}$ becomes $T_z \otimes I$. Careful analysis now shows that under the same unitary equivalence, we have

$$V|_{H^\perp} \approx T_z \otimes UP + I \otimes U(I-P), \qquad W|_{H^\perp} \approx T_z \otimes (I-P)U^* + I \otimes PU^*,$$

where U is some unitary and P is some orthogonal projection operator on $l_2(S)$. In fact, for arbitrary U, P one obtains commuting isometries by the above formula and the pair (U, P) is easily seen to be a complete unitary invariant for the pair $(V|_{H^\perp}, W|_{H^\perp})$. It is not hard to see that the analysis above

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