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LIMITS OF SOLUTIONS OF VOLTERRA INTEGRAL EQUATIONS[1]

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Consider the Volterra integral equation

$$\text{(E)} \qquad u(t) = -\int_0^t A(t-\tau)g(u(\tau))\,d\tau + f(t), \qquad t > 0,$$

on a Hilbert space H. $A(t)$ is a family of bounded, linear, selfadjoint operators on H and g is a nonlinear bounded map from H into itself. If $f(t) \longrightarrow f_0(t)$ as $t \longrightarrow \infty$ then

$$(\mathrm{E}_0) \qquad u_0(t) = -\int_0^\infty A(\tau)g(u_0(t-\tau))\,d\tau + f_0(t), \qquad t > 0,$$

will be called a *limit* equation for (E). The following result appears in [7].

THEOREM (MILLER). *Let $H = R^n$. Suppose $A \in L_1(0, \infty)$, $f: R^+ \longrightarrow R^n$ is bounded and uniformly continuous, g is continuous. Let* (E) *have a bounded solution u on R^+. Then there exist a solution u_0 of* (E_0) *and a sequence $t_n \longrightarrow \infty$ such that $u(t + t_n) \longrightarrow u_0(t)$ as $n \longrightarrow \infty$.*

We give a result complementary to Miller's. We give conditions on A and g which guarantee that if (E_0) has a bounded solution then all solutions of (E) tend to u_0 as $t \longrightarrow \infty$.

Our hypotheses are taken from [5]. We assume that g is continuous, bounded with $g(0) = 0$ and that

$$(1) \qquad \langle g(u) - g(v), u - v\rangle \geqslant m\|u - v\|^2 \quad \text{for some } m > 0.$$

We assume that $A \in C^{(2)}[0, \infty)$, $A^{(k)} \in L_1(0, \infty)$, $k = 0, 1, 2$. A also is to satisfy

$$(2) \qquad \langle A(0)u, u\rangle \geqslant \alpha\|u\|^2, \quad \langle \dot{A}(0)u, u\rangle \leqslant -\beta\|u\|^2, \qquad \alpha > 0, \quad \beta > 0,$$

$$(3) \qquad \begin{array}{c}\text{given any } N, \text{ there exists } \delta(N) > 0 \text{ such that} \\ \langle \operatorname{Re} A\hat{\ }(i\eta)u, u\rangle \geqslant \delta(N)\|u\|^2 \quad \text{for all } |\eta| \leqslant N.\end{array}$$

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