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DEFORMING P.L. HOMEOMORPHISMS ON A CONVEX 2-DISK

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1. **The main result.** Let D be a convex disk in R^2 whose boundary is a polygon. By a *triangulation* of D, we mean a (rectilinear) simplicial complex which has D as its underlying space. We shall call a homeomorphism f of D onto D a p.l. *homeomorphism* if there exists a triangulation K of D such that the restriction of f to each simplex σ of K is a linear map of σ into R^2. We shall consider only those p.l. homeomorphisms of D which are pointwise fixed on the boundary of D. In this note, we announce the following result.

THEOREM A. *For each p.l. homeomorphism f of D, there exists a triangulation K of D such that f may be realized by successively moving the vertices of K in a finite number of steps (with the motion being extended linearly to each simplex of K) such that in the process of moving, none of the simplices is allowed to collapse.*

The general problem of deforming a prescribed map of a space into the identity map, or vice versa, in a specific manner has a long history. For the special case of deforming a particular homeomorphism of an n-cell into the identity map through a special class of homeomorphisms, H. Tietze proved as early as 1914 that any homeomorphism of a 2-disk, which is pointwise fixed on the boundary of the disk, can be deformed into the identity map through a family of such homeomorphisms [5]. This result was extended in 1923 for an n-dimensional cell by J. W. Alexander [1]. The technique used by Alexander can in fact be used to show that each p.l. homeomorphism on a polyhedral n-cell, which is pointwise fixed on the boundary of the cell, can be deformed into the identity map through a family of such p.l. homeomorphisms. However, each of the p.l. homeomorphisms of the family requires a different triangulation of the domain space. It is therefore natural to ask whether this

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