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THE LOOP SPACE PROBLEM AND ITS CONSEQUENCES

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0. **Introduction.** One of the key results in the study of the topology of Lie groups is the following theorem of Bott [2]:

THEOREM. *Let G be a simply connected Lie group. Then $H_*(\Omega G; Z)$ is torsion free.*

Bott subsequently coauthored a paper with Samelson [3] which uses this theorem to obtain extensive information about the homotopy and homology of Lie groups. Later, Araki [1] used this result to compute the mod p cohomology of the exceptional groups E_7 and E_8 over the Steenrod algebra. Bott's proof depends heavily on the existence of a differential structure on the Lie group.

Shortly after Bott proved this result, it was conjectured that the integral homology of the loops on a finite simply connected H-space should be torsion free. We resolve this conjecture for odd primes:

THEOREM 1. *Let X be a simply connected finite H-space. Then $H_*(\Omega X; Z)$ has no odd torsion.*

Actually, we prove this result in a much more general setting. Unlike Bott's proof, which relies heavily on the differential structure, our proof is purely homological and can be applied to H-spaces that do not even have the homotopy type of a finite complex.

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1. **Statement of results.** For the remainder of the paper, X will be a two-connected H-space having the homotopy type of a CW complex with finitely many cells in each dimension. Furthermore, p will be an odd prime, and we will assume $QH^{\text{even}}(X; Z_p)$ is finite dimensional and $\beta_1 QH^{\text{even}}(X; Z_p) = 0$.

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