BULLETIN OF THE
AMERICAN MATHEMATICAL SOCIETY
Volume 81, Number 3, May 1975

A COUNTEREXAMPLE IN SHAPE THEORY[1]

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Communicated by W. Wistar Comfort, December 2, 1974

In the process of trying to solve a certain problem in Harmonic analysis (cf. [5]) the author encountered the following shape theory question: If f: $X \longrightarrow Y$ is a surjective map of compact spaces and $f^{-1}(y)$ has trivial shape for each $y \in Y$, then is f necessarily a shape equivalence? We were able to prove that the answer is yes provided Y is finite dimensional, but this was of no use in the problem we were considering.

We discussed this question with R. D. Anderson, who pointed out that the problem has been known to specialists in shape theory (e.g., Anderson, Borsuk, Mardešić) for some time. Anderson has a proof (as yet unpublished) in the case where $\{y: f^{-1}(y)$ is nondegenerate$\}$ is finite dimensional. In case X and Y are both finite dimensional the result is essentially Theorem 11 of Sher's paper [4].

The purpose of this paper is to present a counterexample for the general question. Specifically, if Q is the Hilbert cube, we will construct a compact metric space X and a surjective map f: $X \longrightarrow Q$ such that $f^{-1}(q)$ has trivial shape for all $q \in Q$, but X does not have trivial shape. Since Q has trivial shape, this means that f is not a shape equivalence.

Our example depends heavily on a K-theory result of Adams. In fact, given Adams' result, ours is little more than an observation.

1. **Shape.** Borsuk [2] invented the concept of shape as a substitute for homotopy type when one is dealing with spaces which are not locally nice (also see [3]). We will not define the notion of shape equivalence here. For our purposes it is enough to know the following: If f: $X \longrightarrow Y$ is a continuous map between compact Hausdorff spaces, then f is a shape equivalence if and only if f^*: $[Y, M] \longrightarrow [X, M]$ is bijective whenever M is an ANR. Thus, if H is any contravariant functor from compact spaces to sets which has a

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AMS (MOS) subject classifications (1970). Primary 57A20, 57A60; Secondary 54G20, 54C55.

[1] Research supported by the National Science Foundation under NSF grant #GP-43114X.