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THE RANGE OF A VECTOR MEASURE

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Let X be a real quasi-complete locally convex topological vector space. Let $K \subset X$ be a weakly compact convex and symmetric set such that $0 \in K$.

Let T be an abstract space and S be a σ-algebra of subsets of T. A vector measure is a σ-additive mapping $m: S \longrightarrow X$.

We are concerned with the question whether there exists a vector measure $m: S \longrightarrow X$ such that K coincides with the closed convex hull of the range of m, i.e. $K = \overline{\mathrm{co}}\, m(S) = \overline{\mathrm{co}}\{m(E): E \in S\}$. The case $X = R^n$ was surveyed in [1].

THEOREM 1. *If T is a space, S a σ-algebra of subsets of T and $m: S \longrightarrow X$ a vector measure, then there exists a space T_1, a σ-algebra S_1 of subsets of T_1 and a vector measure $m_1: S_1 \longrightarrow X$ such that*

$$\overline{\mathrm{co}}\, m(S) = \overline{\mathrm{co}}\, m_1(S_1) = \left\{\int_{T_1} f\, dm_1 : 0 \leqslant f \leqslant 1, f \text{ is } S_1\text{-measurable}\right\}$$

$$= \{m_1(E): E \in S_1\} = m_1(S_1).$$

It is worth mentioning that the equality $\overline{\mathrm{co}}\, m(S) = \{\int f\, dm: 0 \leqslant f \leqslant 1\}$ does not hold, in general [3].

LEMMA. *If $K = \overline{\mathrm{co}}\, m(S)$ and $y \in K$, then there exists a vector measure $m_1: S \longrightarrow X$ such that $K - y = \overline{\mathrm{co}}\, m_1(S)$.*

In view of Theorem 1, the proof of this Lemma is not different from one given by Halmos in the case $X = R^n$ (see [1]). The Lemma permits us to restrict our attention to sets having 0 for the center of symmetry.

Assume that 0 is the center of symmetry of K. For any element $x' \in X'$, the continuous dual of X, let $\|x'\|_K = \sup\{|\langle x', x\rangle|: x \in K\}$.

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