

THE CAUCHY PROBLEM FOR A FIRST ORDER SYSTEM OF ABSTRACT OPERATOR EQUATIONS

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1. **Introduction.** Recently, Carroll [1], [2], and Hersh [5] have studied questions of representation, existence and uniqueness of solutions for the operator differential equation $Q(d/dt, A)u(t) = 0$, where $Q(r, s)$ is a polynomial in r and s and A is a group generator. Their uniqueness results for the Cauchy problem for this equation involve the additional assumption that A^* also generates a group. Earlier results of Hille [6] for the equation

$$(\Delta) \quad [(d^2/dt^2) - A^2] u(t) = 0$$

suggests that this assumption is not necessary for uniqueness. For this equation he shows that a necessary and sufficient condition for the Cauchy problem for (Δ) to have a unique solution is that A generates a group.

In this research announcement we describe a new method for studying questions of uniqueness, representation, and existence for a Cauchy problem for a first order system of differential equations with operator coefficients. This method which employs Schwartz vector valued distributions and the Fourier transformation provides rather complete results for this class of first order systems.

2. **The first order system.** Let $P(s, t)$ be an $n \times n$ matrix whose elements are polynomials in s with coefficients depending continuously upon the parameter t ; let A be the generator of a strongly continuous group $G_A(x)$, $-\infty < x < \infty$, of bounded linear operators on the Banach space Y ; and let $\vec{u}(t)$ be an n -vector whose components are Y -valued functions of t for $t \geq 0$. We con-

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