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H^p SPACES AND EXIT TIMES OF BROWNIAN MOTION[1]

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Let R be a region of the complex plane, Z a complex Brownian motion starting at a point in R, and τ the first time Z leaves R:

$$\tau(\omega) = \inf\{t > 0\colon Z_t(\omega) \notin R\}.$$

There are several ways to study such exit times. Here we describe a new approach that gives rather precise information about the moments of τ.

We shall always assume for simplicity that R contains the origin and Z starts there: $Z_0(\omega) = 0$, $\omega \in \Omega$, where $(\Omega, \mathcal{A}, P)$ is the underlying probability space. If F is a function analytic in the open unit disc D, let

$$\|F\|_{H^p} = \sup_{0<r<1} \left[\int_0^{2\pi} |F(re^{i\theta})|^p \, d\theta\right]^{1/p}$$

and

$$\|\tau^{1/2}\|_p = (E\tau^{p/2})^{1/p}.$$

THEOREM 1. *Suppose R is the range of a function F analytic and univalent in D with $F(0) = 0$. Then, for $0 < p < \infty$,*

$$c_p\|\tau^{1/2}\|_p \leqslant \|F\|_{H^p} \leqslant C_p\|\tau^{1/2}\|_p. \tag{1}$$

In particular,

$$\tau^{1/2} \in L^p(\Omega, \mathcal{A}, P) \Leftrightarrow F \in H^p(D).$$

If R is simply connected and has a nondegenerate boundary, then such a function F exists by the Riemann mapping theorem.

In (1), as elsewhere in this note, the choice of the positive real numbers c_p and C_p depends only on p.

The right-hand side of (1) is true in a more general setting. Let Φ be a continuous nondecreasing function on $[0, \infty]$ with $\Phi(0) = 0$ and $\Phi(2\lambda) \leqslant \gamma\Phi(\lambda)$, $\lambda > 0$.

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