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THE MAXIMUM SIZE OF AN INDEPENDENT SET
IN A NONPLANAR GRAPH

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Suppose G is a simple graph with V vertices that embeds on S_n, a surface of genus n. A set of vertices H is independent in G if no pair of vertices in H is adjacent in G. Let $\alpha(G)$ be the maximum number of vertices in any independent set and $\mu(G)$ be $\alpha(G)/V$, the *independence ratio*.

Set

$$U(n) = \{\mu(G): G \text{ embeds on } S_n\}$$

and

$$L(n) = \{\text{limit points of } U(n)\}.$$

REMARK. It is known that $U(0) \subset (2/9, 1]$ [1], [2] and $U(n) \subset [1/\chi, 1]$ where χ is the Heawood number of S_n $(n \geqslant 1)$ [4]. Erdős [3] has asked if $U(0) \subset [1/4, 1]$, a result implied by the four color conjecture.

CONJECTURE. $L(n) = L(0)\ \forall n$.

THEOREM 1. $L(n) \subset [1/5, 1]\ \forall n$.

In the proof of Theorem 1 the small elements of $U(n)$ are examined and shown to be isolated points with relatively few graphs corresponding to each point. Thus we attempt to characterize those graphs with small independence ratios. In each of the following we assume that G embeds on S_n, $n \geqslant 1$, and