

8. ———, *Contract law and the value of a game*, Israel J. Math. **5** (1967), 135–144. MR **39** #7947.
9. W. F. Lucas, *The proof that a game may not have a solution*, Trans. Amer. Math Soc. **137** (1969), 219–229. MR **38** #5474.
10. J. von Neumann and O. Morgenstern, *Theory of games and economic behavior*, Princeton Univ. Press, Princeton, N.J., 1944. MR **6**, 235.
11. R. Selten, *Valuation of n -person games*, Advances in Game Theory, Princeton, N.J., 1964, pp. 577–626. MR **29** #1081.
12. L. S. Shapley, *A value for n -person games*, Contributions to the Theory of Games, vol. 2, Princeton Univ. Press, Princeton, N.J., 1953, pp. 307–317. MR **14**, 779.
13. ———, *A solution containing an arbitrary closed component*, Contributions to the Theory of Games, vol. 4, Princeton Univ. Press, Princeton, N.J., 1959, pp. 87–94. MR **22** #635.
14. ———, *Values of large market games: Status of the problem*, RM-3957-PR, The Rand Corporation, Santa Monica, 1964.

Convolution equations and projection methods for their solution, by I. C. Gohberg and I. A. Fel'dman, American Mathematical Society Translations, vol. 41, 1974, ix + 261 pp.

Suppose \mathfrak{B}_1 and \mathfrak{B}_2 are Banach spaces, $\{P_\tau\}$ and $\{Q_\tau\}$ are families of projection operators on \mathfrak{B}_1 and \mathfrak{B}_2 respectively which converge strongly as $\tau \rightarrow \infty$ to the respective identity operators, and A is a bounded linear transformation from \mathfrak{B}_1 to \mathfrak{B}_2 . One says that the projection method (P_τ, Q_τ) is applicable to A if, roughly speaking, $(Q_\tau A P_\tau)^{-1}$ converges strongly to A^{-1} as $\tau \rightarrow \infty$. More precisely what is required is that $Q_\tau A P_\tau$, as an operator from $P_\tau \mathfrak{B}_1$ to $Q_\tau \mathfrak{B}_2$, be invertible for sufficiently large τ and that $(Q_\tau A P_\tau)^{-1} Q_\tau$ converge strongly as $\tau \rightarrow \infty$. (Then A is necessarily invertible and the strong limit is A^{-1} .)

To give an example, the prototype of those considered in this book, let a be a bounded function defined on the unit circle having Fourier coefficients a_k ($k=0, \pm 1, \dots$), and consider the operator A on l_2 of the positive integers defined by

$$A\{\xi_j\} = \left\{ \sum_{k=1}^{\infty} a_{j-k} \xi_k \right\}_{j=1}^{\infty}.$$

This is the (semi-infinite) Toeplitz operator associated with a . The projections are the simplest ones: $P_n = Q_n =$ projection on the subspace of sequences $\{\xi_j\}$ satisfying $\xi_j = 0$ for $j > n$. The operator $P_n A P_n$ may then be represented by the finite Toeplitz matrix

$$A_n = (a_{j-k})_{j,k=1}^n$$

and the question is whether these matrices are invertible from some n onward and, if so, whether the inverses of these matrices converge strongly