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Suppose  $\mathfrak{B}_1$  and  $\mathfrak{B}_2$  are Banach spaces,  $\{P_\tau\}$  and  $\{Q_\tau\}$  are families of projection operators on  $\mathfrak{B}_1$  and  $\mathfrak{B}_2$  respectively which converge strongly as  $\tau \rightarrow \infty$  to the respective identity operators, and  $A$  is a bounded linear transformation from  $\mathfrak{B}_1$  to  $\mathfrak{B}_2$ . One says that the projection method  $(P_\tau, Q_\tau)$  is applicable to  $A$  if, roughly speaking,  $(Q_\tau A P_\tau)^{-1}$  converges strongly to  $A^{-1}$  as  $\tau \rightarrow \infty$ . More precisely what is required is that  $Q_\tau A P_\tau$ , as an operator from  $P_\tau \mathfrak{B}_1$  to  $Q_\tau \mathfrak{B}_2$ , be invertible for sufficiently large  $\tau$  and that  $(Q_\tau A P_\tau)^{-1} Q_\tau$  converge strongly as  $\tau \rightarrow \infty$ . (Then  $A$  is necessarily invertible and the strong limit is  $A^{-1}$ .)

To give an example, the prototype of those considered in this book, let  $a$  be a bounded function defined on the unit circle having Fourier coefficients  $a_k$  ( $k=0, \pm 1, \dots$ ), and consider the operator  $A$  on  $l_2$  of the positive integers defined by

$$A\{\xi_j\} = \left\{ \sum_{k=1}^{\infty} a_{j-k} \xi_k \right\}_{j=1}^{\infty}.$$

This is the (semi-infinite) Toeplitz operator associated with  $a$ . The projections are the simplest ones:  $P_n = Q_n =$  projection on the subspace of sequences  $\{\xi_j\}$  satisfying  $\xi_j = 0$  for  $j > n$ . The operator  $P_n A P_n$  may then be represented by the finite Toeplitz matrix

$$A_n = (a_{j-k})_{j,k=1}^n$$

and the question is whether these matrices are invertible from some  $n$  onward and, if so, whether the inverses of these matrices converge strongly