Values of non-atomic games, by R. J. Aumann and L. S. Shapley, Princeton University Press, Princeton, New Jersey, 1974, xi+333 pp., $14.50

Nonatomic games are games played not by a set of individuals but by a measurable space whose measurable sets are called *coalitions*. They are intended as models for economic problems in large populations. Evidently a case could be made—though neither the book nor this review proposes to—that nonatomic games are more fundamental for economic theory than n-player games. No doubt, actual populations are finite; but that is true also of the atoms in a continuous medium.

This is the first book on nonatomic games, and all of the literature in the area is within the conceptual framework that Aumann and Shapley have established. In particular, it is all on values. As the authors say, "an operator that assigns to each player of a game a number that purports to represent what he would be willing to pay to participate $\cdots$ is called a *value*. Value theory for finite games—i.e., n-player games with n finite—was first studied by Shapley **[12]**, and is by now a well established branch of game theory. It is the purpose of this book to develop a corresponding theory for nonatomic games."

The fact is, thirty years after the first book on n-player game theory **[10]**, that none of its major concepts except that of value seems fit to extend to infinity. The Shapley value is wholly noncontroversial—that is, if we understand it in a suitably narrow sense. Aumann and Shapley do stay within that sense in this book. It is as follows. The process of finding a value for a game is commonly split into two stages: finding a characteristic function v, and passing on to a value φv. When Shapley introduced the operator $v \mapsto \varphi v$ **[12]**, the von Neumann-Morgenstern definition of characteristic function **[10]** was the only one available. As the literature on values grew, Shapley published a footnote **[14]** saying that Harsanyi's characteristic function ([3] or [4]) "is to be preferred" in valuation theory. Not all agree **[8]**. But every two-stage evaluation of this type follows Shapley from v to φv, and the principal one-stage evaluation **[11]** agrees in its conclusion with Harsanyi and Shapley—it is intended only to deepen the theoretical basis.

In this book, the game is given as a characteristic function, almost always a real-valued function of bounded variation on the Borel sets of an interval (or an isomorphic measurable space I). Evaluating such a function is the same sort of problem as integrating a suitable function, or for a closer parallel, associating a measure to an outer measure. The initial source of ideas is different, and of course one may profitably revisit the source, but the source does not surround the theory

As for the major concepts of game theory untouched in this book,