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BOOK REVIEWS

Strong rigidity of locally symmetric spaces, by G. D. Mostow, Princeton University Press (Annals of Mathematics Studies, No. 78) 1973, v+195 pp., $7.00

This monograph is primarily devoted to a proof of the following fundamental theorem (which for the sake of simplicity the reviewer states in less than its fullest generality, i.e. for simple groups rather than semisimple groups).

STRONG RIGIDITY THEOREM. *Let G and G' be two simple noncompact connected Lie groups without centers and not locally isomorphic to $\boldsymbol{SL}(2, \boldsymbol{R})$. Suppose $\Gamma \subset G$ and $\Gamma' \subset G'$ are discrete subgroups such that G/Γ and G'/Γ' are compact. Then any isomorphism θ of Γ onto Γ' extends to an isomorphism $\bar{\theta}$ of G onto G'.*

The extension $\bar{\theta}$ is actually unique (see below). (We recall that a connected Lie group is called *simple* if it is nonabelian and has no proper normal subgroup of dimension >0. The lowest dimension of a simple Lie group is 3 and if it is in addition noncompact it is locally isomorphic to $\boldsymbol{SL}(2, \boldsymbol{R})$, the group of 2×2 real matrices with determinant 1.)

In more geometric language, the theorem means that a compact irreducible locally symmetric space (see below) of nonpositive curvature and dimension >2 is uniquely determined up to isometry (and a normalizing factor) by its fundamental group. This theorem has an interesting background which is perhaps best explained by recalling a few facts concerning the exceptional case $\boldsymbol{SL}(2, \boldsymbol{R})$.

If Y is a compact Riemann surface, its homology group $H_1(Y, \boldsymbol{Z})$ has a basis of 1-cycles $a_1, b_1, \cdots, a_g, b_g$ which viewed as loops generate the fundamental group $\pi_1(Y)$ and satisfy the single relation

$$a_1 b_1 a_1^{-1} b_1^{-1} \cdots a_g^{-1} b_g^{-1} = 1. \tag{1}$$

The integer g is called the *genus*. In particular, two Riemann surfaces of the same genus have isomorphic fundamental groups. Let Y and Y' be two such Riemann surfaces of genus >1. As a consequence of uniformization theory, Y and Y' can be written as orbit spaces

$$Y = \Gamma\backslash X, \qquad Y' = \Gamma'\backslash X \tag{2}$$

where X is the upper half-plane, Γ and Γ' are discrete subgroups of the group $G = \boldsymbol{SL}(2, \boldsymbol{R})/\pm 1$ of conformal homeomorphisms of X. The groups Γ and Γ' have no fixed point and are isomorphic to the fundamental