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MEASURES AS CONVOLUTION OPERATORS ON HARDY AND LIPSCHITZ SPACES

BY MISHA ZAFRAN[1]

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In this note we announce some new results concerning the spectral theory of measures as convolution operators. To state our principal theorem, we introduce the following notation. If X is a Banach space and T is a bounded linear operator on X, we write $\mathrm{sp}(T, X)$ to denote the spectrum of T on X. Let G be an LCA group with dual group Γ. $M(G)$ will denote the class of finite regular Borel measures on G, and $M_0(G) = \{\mu \in M(G) \mid \hat{\mu}$ vanishes at infinity on $\Gamma\}$. For $\mu \in M(G)$, let T_μ denote the operator defined by $T_\mu(f) = \mu * f$, that is, convolutions with μ. Finally, let H^1 be the natural domain of the Hilbert transform on $L_1(\mathbf{R})$, and let Lip α denote the usual class of bounded functions on $\mathbf{R}$ satisfying a Lipschitz condition of order α, $0 < \alpha < 1$. We can now state our main result.

THEOREM 1. *There exists a measure* $\mu \in M_0(\mathbf{R})$ *such that*
(a) $\mathrm{sp}(T_\mu, H^1) \neq \hat{\mu}(\mathbf{R}) \cup \{0\}$, *and*
(b) $\mathrm{sp}(T_\mu, \mathrm{Lip}\, \alpha) \neq \hat{\mu}(\mathbf{R}) \cup \{0\}$, $0 < \alpha < 1$.

This may be viewed as an analogue of the now classical Wiener-Pitt theorem concerning the invertibility of Fourier-Stieltjes transforms [4, Theorem 5.3.4]. Moreover, an elementary interpolation argument shows that if $1 < p < \infty$,

$$\mathrm{sp}(T_\nu, L_p) = \hat{\nu}(\mathbf{R}) \cup \{0\},$$

for all $\nu \in M_0(\mathbf{R})$ (see [1, §1.4]). Thus, in a sense, our theorem is intermediate between the L_1 and L_p $(1 < p < \infty)$ cases.

The proof of Theorem 1 is based on the following result.

THEOREM 2. *Let*

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