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ON RELATIVE CLASS NUMBERS OF CERTAIN QUADRATIC EXTENSIONS[1]

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1. **Introduction.** As a generalization of the well-known class number formula of an algebraic number field, we obtain in [3] a formula for the class number of an arbitrary algebraic torus defined over the rational number field $\mathbf{Q}$. With this generalized class number formula, the relative class number of two isogenous tori can be expressed in terms of their Tamagawa numbers and certain indices of maps induced naturally by an isogeny between them. To be specific, let $\lambda: T \longrightarrow T'$ be an isogeny of tori defined over $\mathbf{Q}$. The isogeny induces naturally the following maps (cf. [3]):

$$\lambda_v: T_v \longrightarrow T'_v, \quad \lambda_v^c: T_v^c \longrightarrow T'^c_v, \quad \lambda_{\mathbf{Q}}^\infty: T_{\mathbf{Q}}^\infty \longrightarrow T'^\infty_{\mathbf{Q}}, \quad (\hat{\lambda})_{\mathbf{Q}}: (\hat{T}')_{\mathbf{Q}} \longrightarrow (\hat{T})_{\mathbf{Q}}.$$

For a homomorphism $\alpha: G \longrightarrow G'$ of commutative groups with finite kernel and cokernel, we define the q-symbol of α by $q(\alpha) = [\mathrm{Cok}\,\alpha]/[\mathrm{Ker}\,\alpha]$. Then, the q-symbols of the above maps are finite, and $q(\lambda_v^c) = 1$ for almost all v (cf. [3]). The relative class number $h_T/h_{T'}$ of T, T' over $\mathbf{Q}$ can now be expressed as (cf. [3]):

$$\frac{h_T}{h_{T'}} = \frac{\tau_T}{\tau_{T'}} \cdot \frac{q(\lambda_\infty)}{q(\lambda_{\mathbf{Q}}^\infty)q((\hat{\lambda})_{\mathbf{Q}})} \cdot \prod_{v \neq \infty} q(\lambda_v^c). \tag{1}$$

In this paper, we apply (1) to the study of relative class numbers of certain quadratic extensions of algebraic number fields.

2. **Relative class numbers.** Let $k/\mathbf{Q}$ be a finite extension, and K/k be a Galois extension of finite degree n. Denote by N the norm map $R_{K/k}(\mathbf{G}_m) \longrightarrow \mathbf{G}_m$, where $R_{K/k}$ is the Weil functor of restricting the field of definition (cf. [4]), and $\mathbf{G}_m$ is the multiplicative group of the universal domain. We have an exact sequence (N) $0 \longrightarrow \mathrm{Ker}\, N \xrightarrow{i} R_{K/k}(\mathbf{G}_m) \xrightarrow{N} \mathbf{G}_m \longrightarrow 0$ of

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[1] This paper is based on a part of the author's Ph.D. thesis written at Johns Hopkins University. For the unexplained notions, see [3].