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A 4-MANIFOLD WHICH ADMITS NO SPINE

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1. This note is to present a new example which reveals the impossibility of embedding a 2-torus in a 4-manifold.

THEOREM 1. *There exists a compact 4-dimensional PL manifold* W^4 *with boundary satisfying the following conditions*: (i) W^4 *is homotopically equivalent to the 2-torus* $T^2 = S^1 \times S^1$, *and* (ii) *no homotopy equivalence* $T^2 \longrightarrow W^4$ *is homotopic to a PL embedding.*

By a PL embedding is meant one which is not necessarily locally flat.

Theorem 1 is an application of the codimension two surgery theory developed in our previous papers [4], [5], [6]. The phenomena of "total spinelessness" in higher dimensions (with finite π_1's) were found by Cappell and Shaneson [2] using another method of surgery[2] [1].

A calculation in our proof leads to another consequence concerned with submanifolds in codimension two. Let K^{4n} denote a product $CP_2 \times \cdots \times CP_2$ of n-copies of the complex projective plane CP_2.

THEOREM 2. *For each* $n \geqslant 0$, *there exists a locally flat embedding* $h_{(4n)}$ *of* $K^{4n} \times S^1$ *into the interior of* $K^{4n} \times D^2 \times S^1$, *which is homotopic to the zero cross section* $K^{4n} \times \{0\} \times S^1$, *but is not locally flatly concordant to a splitted embedding.*

A *splitted embedding* (with respect to a point $*$ of S^1) means a locally flat embedding $f: K^{4n} \times S^1 \longrightarrow K^{4n} \times D^2 \times S^1$ such that (i) f is transverse regular to $K^{4n} \times D^2 \times \{*\}$ so that the intersection $M^{4n} = f(K^{4n} \times S^1) \cap K^{4n} \times D^2 \times \{*\}$ is a closed manifold, and (ii) the inclusion $M^{4n} \longrightarrow K^{4n} \times D^2 \times \{*\}$ is a homotopy equivalence.

Theorem 2 contrasts with Farrell and Hsiang's result [3] which may be

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[2] Their theory (with Γ-groups) and ours (with P-groups) are not the same but both admit a more general unifying algebraic treatment [7].