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THE STABILITY PROBLEM IN SHAPE AND A WHITEHEAD THEOREM IN PRO-HOMOTOPY

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1. **Shape.** We give a solution to the following

PROBLEM. *Give necessary and sufficient conditions for a compactum Z to have the shape of* (A) *a complex or* (B) *a finite complex.*

Problem B makes sense in Borsuk's shape theory for compacta [2] but in order to give meaning to Problem A, we must extend Borsuk's theory to include noncompact complexes. A particularly simple treatment is in [7]. Alternatively one can replace "complex" by "ANR" in Problem A, and use Fox's extension to metric spaces [9].

It is desirable that the conditions in Problems A and B be intrinsic. The following partial solution to Problem B is in [10]: *a finite-dimensional* 1-*UV compactum has the shape of a finite complex if and only if its Čech cohomology with integer coefficients is finitely generated.* But without the hypothesis 1-*UV*, the condition offered in [10] is not an intrinsic one.

Now for our solution. First some notation. If (Z, z) is a pointed connected compact subset of a euclidean space E, let $\{(X_\alpha, z)\}$ be the inverse system of all connected open neighborhoods of Z in E, pointed by z and bonded by inclusion. Regarding $\{(X_\alpha, z)\}$ as an object of pro-H_0 [1] let pro-$\pi_k(Z, z)$ be the pro-group $\{\pi_k(X_\alpha, z)\}$; let $\check{\pi}_k(Z, z)$ be its inverse limit (the kth shape group of (Z, z)). Let $\widetilde{K}^0(G)$ denote the reduced projective class group of the group G (see p. 64 of [12]).

THEOREM 1 [8]. *Let* (Z, z) *be as above. The following are equivalent*: (i) *pro*-$\pi_k(Z, z)$ *is isomorphic to* $\check{\pi}_k(Z, z)$ *in pro-groups for each* $k \geqslant 1$; (ii) (Z, z) *has the pointed shape of a pointed complex of dimension* $\max\{3, \dim Z\}$; (iii) (Z, z) *is dominated in pointed shape by a pointed finite complex*; (iv) (Z, z) *is movable and the natural topology on* $\check{\pi}_k(Z, z)$ *is discrete for each* $k \geqslant 1$; (v) (Z, z) *is a pointed FANR. Furthermore, Z has the shape*

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