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BOOK REVIEW

Foundations of a structural theory of set addition, by G. A. Freiman, American Mathematical Society, Translations of Mathematical Monographs, vol. 37, 1973, vii+108 pp., $15.70

This is an advanced monograph addressed primarily to experts in additive number theory. Although the only formal prerequisite is a basic knowledge of elementary number theory, some of the proofs are rather sophisticated, and while the author states that he has presented the material of the book to teacher trainees, this would be quite unthinkable in the United States.

There are three chapters entitled *Isomorphisms*, *The fundamental theorem on sums of finite sets*, and *Sums of sequences, sets of residues and point sets*. In the first chapter, which is by far the most clearly written of the three, the author introduces some basic concepts and formulates the problems to be attacked later in the book. Suppose that A and B are two sets with a binary operation, denoted in both cases by $+$. (The operations are not assumed to be associative or commutative.) Two subsets $A' \subseteq A$, $B' \subseteq B$ are called *isomorphic* if there is a bijection $a \leftrightarrow b$ from A' to B' such that $a_1+a_2=a_3+a_4$ if and only if $b_1+b_2=b_3+b_4$. For example, if $A=\boldsymbol{Z}$ and $B=\boldsymbol{Z}^2$ (where $\boldsymbol{Z}^n$ denotes the group of lattice points in n-dimensional Euclidean space $\boldsymbol{R}^n$), then the subsets

$$A' = \{0, 1, 3, 4\} \quad \text{and} \quad B' = \{(0, 0), (1, 0), (0, 1), (1, 1)\}$$

are isomorphic. A rather grandiose statement is made to the effect that additive number theory is the study of properties invariant under isomorphism, and comparisons with Klein's *Erlanger Programm* are drawn. The reviewer finds this quite ludicrous, but it does enliven some otherwise dull moments in reading the book.

Now let K be a set of finite cardinality $|K|=k$, and $2K$ the set of all sums $a+b$, where $a, b \in K$. The author distinguishes between *direct* problems of additive number theory, where properties of $2K$ are deduced from those of K, and *inverse* problems, where properties of K are deduced from those of $2K$. The present book is primarily concerned with inverse problems, and in particular with determining the structure of K under the hypothesis that the cardinality $|2K|=T$ is small. As an example of a direct result we may take the Cauchy-Davenport theorem, which asserts that if K is a