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ASYMPTOTIC ANALYSIS OF TRANSPORT PROCESSES

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ABSTRACT. For a large class of processes called transport processes we study in detail a certain asymptotic limit, the diffusion limit. Transport processes arise in linear transport theory, learning theory, nonlinear oscillations in the presence of noise and other problems. We examine closely some examples and give complete proofs for the results stated here.

1. **Introduction.** The motion of a particle whose velocity undergoes jumps of random size at random times constitutes the prototype of a transport process. The general structure of transport processes however underlies a great variety of problems that arise in mathematical sciences and which may have no relation to the moving particle model. Our aim here is to study the properties of a large class of transport processes in a specific asymptotic limit, the diffusion limit. This limit corresponds roughly to the frequency of jumps becoming very large in the particle model. We give several examples to illustrate the scope of the asymptotic theory. We also give complete proofs for all results stated here. The contents are briefly as follows.

In §2 we outline the probabilistic construction of transport processes. The general theory of Markov processes [1], [2], [3] provides the necessary existence, uniqueness, and related information, so it is only sketched here. We present in detail, however, the connection of transport processes with linear transport theory [4]. In §3 we formulate the asymptotic problem that concerns us here by introducing a small parameter that corresponds to the mean free time between jumps. Diffusion approximations are well known in a variety of contexts [4], [5], [6] but for transport processes in the generality of our formulation many special features arise and the treatment of diverse problems is unified. The special features account for the versatility of the asymptotic theory. §4 illustrates this point. We consider a mathematical model of learning theory [7], [8], and show that some

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