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ON LOCAL SOLVABILITY OF LINEAR PARTIAL DIFFERENTIAL EQUATIONS NOT OF PRINCIPAL TYPE

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I. **Introduction.** Necessary and sufficient conditions have been found [5], [6], [7] for the local solvability of linear partial differential operators of principal type. An operator $P(x, D)$ of order m on an open domain $\Omega \subset \mathbf{R}^N$ is said to be of principal type in Ω if $P_m(x, \xi) = 0, x \in \Omega$, $\xi \in \mathbf{R}^N \sim \{0\}$ implies that $\nabla_\xi P_m(x, \xi) \neq 0$. L. Nirenberg and F. Treves [6], [7] have shown that if

(i) $P_m(x, D)$ has analytic coefficients,

(ii) for all complex numbers z, $\operatorname{Im}(zP_m)$ does not change signs in Ω along any null-bicharacteristic strip of $\operatorname{Re}(zP_m)$,

then $P(x, D)$ is locally solvable in Ω. Hereafter we shall say that an operator $P(x, D)$ of principal type satisfies the N-T (Nirenberg-Treves) condition if $P_m(x, D)$ satisfies conditions (i) and (ii) above.

We remark here that for operators of principal type, local solvability depends only upon the leading terms. By contrast, for operators not of principal type one must consider lower order terms. Similar considerations arise in determining the hyperbolicity of operators with multiple real roots in their principal parts [1], [3], [4].

With the above remarks in mind, we specify our problem. Let $P_m(x, D) = Q_1^{j_1}(x, D) \circ \cdots \circ Q_k^{j_k}(x, D)$ with

(i) Each $Q_i(x, D)$ a homogeneous operator of principal type which satisfies the N-T condition.

(ii) $Q_i^{j_i}(x, D) = Q_i(x, D) \circ \cdots \circ Q_i(x, D)$ j_i-times.

(iii) The $Q_i(x, \xi)$'s having no common real roots except for $\xi = 0$.

We state the following definition.

DEFINITION. An operator $T(x, D)$ of order $l < m$ is an admissible lower order perturbation of $P_m(x, D)$ if $\forall b \in C^\infty(\Omega), P_m + bT$ is locally solvable in Ω.

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