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ULTRAFILTERS AND ALMOST DISJOINT SETS. II

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Unless otherwise stated, κ is an arbitrary infinite regular cardinal. For every infinite cardinal κ, $\mu\kappa$ is the family of uniform ultrafilters on κ. Our main result is:

THEOREM 1. *Suppose that* $2^\kappa = \kappa^+$. *Then for every* $U \in \mu\kappa$ *there is a family* $\{a_x: x \in U\}$ *such that: for every* $x \in U$, $a_x \subseteq x$ *and* $|a_x| = \kappa$; *and for every* $x, y \in U$ *with* $x \neq y$, $|a_x \cap a_y| < \kappa$.

This answers a question of Comfort communicated privately to the author and partially answers a question of Hindman [5]. It is still open whether Theorem 1 holds for singular κ as well. The hypothesis $2^\kappa = \kappa^+$ cannot be outright removed, since by [1] it is consistent with ZFC that there is no $A \subseteq P(\kappa)$ such that $|A| = 2^\kappa$, $|a| = \kappa$ for every $a \in A$, and $|a \cap b| < \kappa$ for every $a, b \in A$ with $a \neq b$. See [4], [5] and [8] for other relevant results.

DEFINITION 1. For $A \subseteq P(\kappa)$ and ideal $I \subseteq P(\kappa)$, I is said to be dense in A modulo sets of power $< \kappa$ if for each $x \in A$ such that $|x| = \kappa$, there is some $y \in I$ with $y \subseteq x$ and $|y| = \kappa$. For brevity we shall write "I is dense in A mod($< \kappa$)". I is dense mod($< \kappa$) if I is dense in $P(\kappa)$ mod($< \kappa$).

"I is λ-complete" is defined as in [7].

THEOREM 2. *For every* $U \in \mu\kappa$ *there is a* κ*-complete ideal* $I \subseteq P(\kappa) - U$ *such that* I *is dense* mod($<\kappa$).

Theorem 1 follows from Theorem 2 by induction on ordinals $< \kappa^+$, $a_x (x \in U)$ being chosen to belong to I. See [8, Theorem 1].

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