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GROUPS OF DIFFEOMORPHISMS OF R^n AND THE FLOW OF A PERFECT FLUID

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Introduction. In this paper it is shown how groups of diffeomorphisms can be used to give a new proof of the short-time existence and uniqueness of solutions to the Euler equations for a perfect fluid over R^n:

$$\text{(E)} \qquad \partial U_t/\partial t + DU_t \cdot U_t = -\text{grad}\, P_t, \qquad \text{div}\, U_t = 0.$$

T. Kato [4] and H. Swann [6] achieved similar results by showing that solutions to the Navier-Stokes equations for viscous flows converge to solutions of E in $H^s = L_s^2$ as the viscosity approaches zero. Our results differ from theirs in that the solutions we establish possess a wider variety of asymptotic conditions at infinity.

The proofs of the theorems shall appear elsewhere [1], [2].

Future work includes the extension of these results to flows over non-compact Riemannian manifolds and the study of the related problem of exterior flows in R^n.

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1. **Groups of diffeomorphisms.** We use the standard multi-index notation for differential operators. $\sigma(x) = (|x|^2 + 1)^{1/2}$ and JF denotes the Jacobian determinant.

DEFINITION 1.1. Let $|\ |_p$ be the standard L^p norm on R^n. Then define

$$\|f\|_{o,p,\delta} \doteq |\sigma^\delta \cdot f|_p$$

and

$$\|f\|_{s,p,\delta} = \sum_{|\alpha| \leq s} \|D^\alpha f\|_{o,p,\delta+|\alpha|}.$$

Also, define $M^p_{s,\delta}(R^n, R^m)$ to be the completion of $C_0^\infty(R^n, R^m)$ with re-

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