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UNIFORMLY TRIVIAL MAPS INTO SPHERES

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A map (continuous function) is uniformly trivial if it is uniformly homotopic to a constant map. The universal covering $e: R \longrightarrow S^1$ of the circle by the real line is an example of a map which is homotopically trivial but not uniformly so. For any space X a map $f: X \longrightarrow S^1$ is uniformly trivial if and only if there is a bounded map $g: X \longrightarrow R$ such that $eg = f$ [E].

For spheres of higher dimension it has been shown that every map from euclidean n-space or $(n + 1)$-space to the n-sphere, S^n, is uniformly trivial [C1], [C2]. Here we announce the following extensions of these results.

THEOREM 1. *For (X, A) a finite dimensional triangulable pair of spaces and $n > 1$, a map $f: (X, A) \longrightarrow (S^n, *)$, $* \in S^n$, is uniformly trivial if and only if it is homotopically trivial.*

THEOREM 2. *For X a contractible finite dimensional triangulable space and Y a compact space, the fundamental group, $\pi_1(Y)$, of Y being finite implies that every map from X to Y is uniformly trivial, but there exists uniformly nontrivial maps from X to Y if X is noncompact and $\pi_1(Y)$ contains an element of infinite order.*

If (X, A) and (Y, B) are pairs of completely regular hausdorff spaces, the two maps $f, g: (X, A) \longrightarrow (Y, B)$ are *uniformly homotopic* if their extensions $\beta f, \beta g: (\beta X, \beta A) \longrightarrow (\beta Y, \beta B)$ are homotopic. Here β denotes the Stone-Čech compactification. For equivalent definitions see [D] and [ES].

OUTLINE OF PROOF OF THEOREM 1. For $n \geqslant 1$ the free topological group F on S^n can be considered as a CW-complex of finite type (i.e. the m-skeleton F^m of F is compact for each m), and that there is an embedding, $i: S^n \longrightarrow F$, of S^n as a subcomplex of F, which represents a generator of $\pi_n(F)$. This is "folklore" but can easily be deduced from the proof of Theorem 1 in [H].

Let (E, p, S^{n+1}) be the fiber bundle over S^{n+1} with fiber

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