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MAPPING CYLINDER NEIGHBORHOODS OF SOME ANR'S

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Let X be a closed subset of the interior of a manifold Q. A submanifold M of Q is a *mapping cylinder neighborhood* of X if it is a closed neighborhood of X in Q and if there is a proper map $r: \partial M \longrightarrow X$ such that M is homeomorphic to the mapping cylinder of r, fixing ∂M and X in the natural way. Since mapping cylinders strong deformation retract to their targets, a set that possesses a mapping cylinder neighborhood in some manifold is a finite dimensional ANR. Moreover, if the set is compact, its mapping cylinder neighborhood is a compact manifold. It follows from Kirby and Siebenmann [K–S] that the neighborhood, hence the set, has finite homotopy type.

Regular neighborhoods of locally finite complexes in PL manifolds are mapping cylinder neighborhoods. More generally, R. D. Edwards showed [E] that stably, locally finite cell complexes in manifolds have mapping cylinder neighborhoods. We prove the following

THEOREM. *Let M' be a manifold. Suppose $M = M' \cup$* (open outside collar of $\partial M'$), *and that M supports a fixed point free flow whose flow lines give an oriented foliation on M that is transverse and outward pointing on $\partial M'$. Suppose X is a locally compact ANR embedded as a closed, $1 - LC$, codimension 4 subset of the interior of a manifold Q. Then $X \times M'$ has a mapping cylinder neighborhood in $Q \times M$.*

We recover as a corollary the result of Gersten [G] and Wall [W] that the product of a compact, finite dimensional ANR with a compact manifold of Euler characteristic zero has finite homotopy type.

More significant is the case where $M' = (-1, 1]$ and $M = (-1, \infty)$. It suggests the possibility of joining two copies of a mapping cylinder neighborhood of $X \times (-1, 1]$ back to back in order to get a mapping cylinder neighborhood of $X \times [-1, 1]$ in some manifold, which easily implies the

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