TRIGONOMETRY ON THE UNIT BALL
OF A COMPLEX HILBERT SPACE

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1. **Introduction.** The unit disc furnished with the Poincaré metric pro-
vides a model (the Poincaré model) for the hyperbolic geometry. The linear
fractional transformations of the unit disc onto itself constitute the group of
motions. An analogous phenomenon arises in the unit ball of any complex
Hilbert space when it is furnished with the Carathéodory-Reiffen metric.

The purpose of this note is to announce the hyperbolic version of the
laws of sines and cosines, and the Pythagorean theorem on the unit ball of any
complex Hilbert space.

2. **Preliminaries.** Let X and Y be complex Banach spaces and let
$D \subset X$ be a domain (= open connected subset of X). A map $f: D \to Y$
is *holomorphic* if the Fréchet derivative of f at each $x \in D$ (denoted by
$Df(x)$) exists and is complex linear. Let G be a domain of Y. A map
$f: D \to G$ is *biholomorphic* if the inverse map $f^{-1}: G \to D$ exists and
both f and f^{-1} are holomorphic. A domain D is *homogeneous* if for
each pair of points x, x' in D there exists a biholomorphic map $f: D \to D$
with $f(x) = x'$.

Let $\Delta(D)$ denote the class of holomorphic maps of D into the unit
disc Δ in the complex plane C. Following [1], we define the Carathéodory-
Reiffen metric by

$$\alpha_D(x, \xi) = \sup \{|Df(x)\xi|: f \in \Delta(D)\}, \quad x \in D, \quad \xi \in X,$$

where $| \; |$ denotes the norm in C.

The distance $\rho_D(x, x')$ between two points x and x' in D is defined
in the usual way. Namely,