

NEW INFINITE FAMILIES IN THE
 STABLE HOMOTOPY OF SPHERES

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Let p be an odd prime and denote by π_*^s the stable homotopy ring of spheres. There are known two infinite families of elements

$$\alpha_t \in \pi_*^{2t(p-1)-1}, \quad \beta_t \in \pi_*^{2t(p^2-1)-2p}$$

introduced in [4] and [2I]; the latter sequence only when $p > 3$. In [5] Toda also constructed elements

$$\epsilon_i \in \pi_*^{2(p-1)(p^2+i)-2} \quad i = 1, \dots, p-1,$$

and by a seemingly numerical accident in Toda's tables (at least up to a non-zero multiple) $\epsilon_{p-1} = \beta_p$. Thus the element ϵ_{p-1} has a bordism theoretic interpretation. In this note we announce the results of our investigation of the remaining elements $\epsilon_1, \dots, \epsilon_{p-2}$. To describe our results we recall some notations and results of [2].

Let $\Omega_*^U(\)$ denote the complex bordism homology theory [3]. Recall that the coefficient ring is

$$\Omega_*^U \simeq \mathbf{Z}[x_2, x_4, \dots]; \quad \deg x_{2i} = 2i$$

an infinite polynomial ring. Polynomial generators of dimension $2p^i - 2$ play a special role in the theory, are called *Milnor manifolds*, and are usually denoted by \mathbf{V}^{2p^i-2} .

Following the stable notations of [2] let $V(0) = S^0 \cup_p e^1$, so that $\tilde{\Omega}_*^U(V(0)) \simeq \tilde{\Omega}_*^U(p)$. There is the map [2I; 1.5] $\bar{\varphi}: S^{2p-2}V(0) \rightarrow V(0)$ such that $\bar{\varphi}_*: \tilde{\Omega}_*^U(S^{2p-2}V(0)) \rightarrow \tilde{\Omega}_*^U(V(0))$ is given by

$$\bar{\varphi}_*(\Sigma^{2p-2}\gamma_0) = [\mathbf{CP}(p-1)]\gamma_0,$$

where $\gamma_0 \in \tilde{\Omega}_0^U(V(0))$ is a generator. There is the iteration $\bar{\varphi}^s: S^{2s(p-1)}V(0) \rightarrow V(0)$ whose mapping cone we denote by $V^{(s)}(1)$. (N.B. when $s = 1$

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