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A CONSTRUCTIVE CHARACTERIZATION OF DISCONJUGACY

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An ordinary linear differential operator L defined by

$$Ly = y^{(n)} + a_{n-1}(t)y^{(n-1)} + \cdots + a_1(t)y' + a_0(t)y \tag{1}$$

is said to be disconjugate on an interval I if every nontrivial solution of

$$Ly = 0 \tag{2}$$

has less than n zeros on I, multiple zeros being counted according to their multiplicity.

We assume $a_i \in C(I)$ for $i = 0, \cdots, n-1$. This assumption is made mainly for convenience and can be considerably weakened.

We announce here an algorithm for the construction of disconjugate operators of type (1) for any $n \geqslant 2$ and all intervals I. Our construction yields all disconjugate operators of type (1) if the interval I is either open or compact. This construction has the following features: It is iterative or inductive in the sense that the set of nth order disconjugate operators is constructed from the set of $(n-1)$st order ones. (The second order from the first order ones. All first order operators $y' + a_0(t)y$ are disconjugate.) The procedure for going from $n-1$ to n involves a parameter function.

For the remainder of this paper I denotes any compact or open interval, $C(I)$, the set of real valued continuous functions on I and $C'(I)$ the set of real valued functions on I which have continuous first derivatives.

THEOREM 1. *Given a_0 in $C(I)$ there exist $a_1, \cdots, a_{n-1}$ in $C(I)$ such that* (1) *is disconjugate. Moreover* (1) *is disconjugate if and only if there exists r in $C(I)$ and $b_0, \cdots, b_{n-2}$ in $C'(I)$ such that:*

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