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COHEN-MACAULAY RINGS AND CONSTRUCTIBLE POLYTOPES

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We wish to point out how certain concepts in commutative algebra are of value in studying combinatorial properties of simplicial complexes. In particular, we obtain new restrictions on the *f*-vectors of simplicial convex polytopes.

Let Δ be a finite simplicial complex with vertex set $V = \{v_1, v_2, \cdots, v_n\}$. We call the elements of Δ the *faces* of Δ. If the largest face of Δ has d elements, then we say $\dim \Delta = d - 1$. The *f-vector* of Δ is $(f_0, f_1, \cdots, f_{d-1})$, where $\dim \Delta = d - 1$ and exactly f_i faces of Δ have $i + 1$ elements. Define for positive integers m,

$$H(\Delta, m) = \sum_{i=0}^{d-1} f_i \binom{m-1}{i}.$$

Also define $H(\Delta, 0) = 1$. We say that Δ is *constructible* [2] if it can be obtained by the following recursive procedure: (a) Every simplex is constructible, and (b) if Δ and Δ' are constructible of the same dimension d, and if $\Delta \cap \Delta'$ is constructible of dimension $d - 1$, then $\Delta \cup \Delta'$ is constructible.

We know of two main classes of constructible Δ's: (A) The boundary complex of a simplicial convex polytope is constructible. This follows from [1]. (B) Let D be a finite distributive lattice, and let D' be D with the top element and bottom element removed. Let Δ be the simplicial complex whose faces are the chains of D'. Then Δ is constructible.

If h and i are positive integers, then h can be written uniquely in the form

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