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BOOK REVIEW

Invariant subspaces, by H. Radjavi and P. Rosenthal, Springer-Verlag, New York, 1973, xi+219 pp., DM 50.—

In recent years, a large number of invariant subspace questions have been raised, and some of them answered, in an effort to make some progress towards structure theorems for operators and algebras of operators. Typically, a collection $\mathscr{A}$ of bounded linear operators on a Hilbert space $\mathscr{H}$ is specified. The lattice of $\mathscr{A}$ is to the collection of all closed linear subspaces M of $\mathscr{H}$ which are invariant under $\mathscr{A}$; that is, $A(M)\subset M$ for each A in $\mathscr{A}$. What is the lattice of $\mathscr{A}$ like? In particular, must the lattice contain more than $\{0\}$ and $\mathscr{H}$ if $\mathscr{A}$ consists of a single operator (the invariant subspace question), or if A is the algebra of operators which commute with a particular operator (the hyperinvariant subspace question), or if A is any algebra of operators (the transitive algebra question)? The book under review is a report on the progress towards some adequate answers to these questions.

The book is carefully organized and sufficiently self-contained to be accessible to most mathematicians. Background material which is not part of a standard graduate course and which is not explicitly developed is at least discussed and referred to. There are no long proofs whose punch lines come out of nowhere. Some results which are pertinent to the subject of this book, but which are parts of elaborate theories, have been omitted. For these results, the reader is referred to specific accounts of these theories by, for example, Sz.-Nagy-Foiaş or Foiaş-Colojoară. (See Bull. Amer. Math. Soc. 77 (1972), 938–942.) At the end of each chapter is an Additional Propositions section of homework problems for the reader, and a Notes and Remarks section in which credit for theorems is given and additional results are mentioned.

The book begins with the usual Chapter 0 in which terminology and notation are set and background material is discussed. The first few real chapters cover standard topics in operator theory, much of which appears elsewhere in book form. Chapter 1 on normal operators gives a nice proof of the spectral theorem which hints at Banach algebras without mentioning them. This chapter also includes an easy proof of Fuglede's theorem based on Rosenblum's study of the operator equations $AX-XB=Y$. Chapter 2 develops the Riesz functional calculus and gives a useful