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SOME CONVEXITY AND SUBADDITIVITY PROPERTIES OF ENTROPY

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I. **Introduction.** Statistical mechanics is the science of explaining, predicting and understanding the gross, macroscopic attributes of matter (which may be taken to mean mechanical systems with essentially an infinite number of degrees of freedom) in terms of the elementary dynamical laws governing its atomic constituents. The problems that arise are sufficiently complex and intriguing, but at the same time sufficiently well posed, that the subject is nowadays as much a part of mathematics as of physics. The fields of information theory and ergodic theory had their genesis in statistical mechanical modes of thought and are now well established in the mathematics literature; there will be more to come.

Ludwig Boltzmann, who died in 1906, was one of the principal founders of statistical mechanics, and his monument in Vienna contains the following eloquent testimonial to his scientific creativity:

$$S = k \log W. \tag{1}$$

Surely, this hypothesis of Boltzmann [**1**] is one of the most important and daring in statistical mechanics, for it relates S, the macroscopic entropy of a system, to W, the number of microscopic states of the system which have the same, given macroscopic properties. The number k is a universal constant, called Boltzmann's constant, and, for our purposes, we can consider it to be 1.

In these lectures we shall explore some of the abstract properties of entropy, after first giving a precise formulation of it, and will include some recent results (with M. B. Ruskai) which extend formerly known facts about the strong subadditivity of entropy from the domain of classical mechanics to the quantum-mechanical domain. The presentation here will be sketchy and the reader is referred to the original papers [**3**], [**4**], [**5**] for more details.

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