

ADAPTIVE QUADRATURE: CONVERGENCE OF PARALLEL AND SEQUENTIAL ALGORITHMS

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Adaptive quadrature algorithms dynamically choose the weights and abscissae in the formula

$$\int_0^1 f(x) dx \approx Q_N f = \sum_{i=1}^N w_i f(x_i)$$

to adapt their estimates to the particular nature of the integrand $f(x)$. Within the past five years experimental evidence has appeared to suggest that adaptive quadrature algorithms are significantly superior to traditional quadrature formulas because they have a much wider domain of efficient applicability with little sacrifice in computational effort.

A metalgorithm is an abstraction representing a large class of algorithms and is used to discuss and analyze the properties of adaptive quadrature algorithms. A novel feature of these algorithms is the important role that data structures (for the interval collection) play in their behavior. A study of reasonable selections of components for the metalgorithm shows that there are from 1 to 10 *million* potentially interesting, significantly distinct adaptive quadrature algorithms. This situation illustrates the extreme difficulty of the common problem of selecting the "best" algorithm for a particular computation.

Our purpose is to announce convergence results for a variety of adaptive quadrature algorithms² (including all 10 million mentioned above). Roughly

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²This announcement is a summary of results and analysis contained in the following:

A metalgorithm for adaptive quadrature, CSD-TR 89, Computer Science Department, Purdue University, March, 1973, 43pp. *J. Assoc. Comput. Mach.* (to appear).

Parallel algorithms for adaptive quadrature: Convergence, CSD-TR 104, Computer Science Department, Purdue University, September, 1973, 18pp. *Proc. IFIP 74* (to appear).

Parallel algorithms for adaptive quadrature II; Metalgorithm correctness, CSD-TR 107, November, 1973, 28pp. (to appear).

Parallel algorithms for adaptive quadrature III; Program correctness, CSD-TR 112, March, 1974, 52pp. (to appear).