ADAPTIVE QUADRATURE: CONVERGENCE OF PARALLEL AND SEQUENTIAL ALGORITHMS

BY JOHN R. RICE

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Adaptive quadrature algorithms dynamically choose the weights and abscissae in the formula

$$\int_0^1 f(x) \, dx \approx Q_N f = \sum_{i=1}^{N} w_i f(x_i)$$

to adapt their estimates to the particular nature of the integrand $f(x)$. Within the past five years experimental evidence has appeared to suggest that adaptive quadrature algorithms are significantly superior to traditional quadrature formulas because they have a much wider domain of efficient applicability with little sacrifice in computational effort.

A metalgorithm is an abstraction representing a large class of algorithms and is used to discuss and analyze the properties of adaptive quadrature algorithms. A novel feature of these algorithms is the important role that data structures (for the interval collection) play in their behavior. A study of reasonable selections of components for the metalgorithm shows that there are from 1 to 10 million potentially interesting, significantly distinct adaptive quadrature algorithms. This situation illustrates the extreme difficulty of the common problem of selecting the "best" algorithm for a particular computation.

Our purpose is to announce convergence results for a variety of adaptive quadrature algorithms$^2$ (including all 10 million mentioned above). Roughly


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$^2$This announcement is a summary of results and analysis contained in the following:


Parallel algorithms for adaptive quadrature: Convergence, CSD-TR 104, Computer Science Department, Purdue University, September, 1973, 18pp. Proc. IFIP 74 (to appear).
