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COMPACT HOLOMORPHIC MAPPINGS ON BANACH SPACES AND THE APPROXIMATION PROPERTY

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1. Let E be a complex Banach space. It is well known that $C(E; C)$, the space of continuous scalar-valued functions on E endowed with the compact-open topology, always has the approximation property, since there are continuous partitions of unity. However, for the space $H(E; C)$ of holomorphic scalar-valued functions on E, the situation is more complicated. In §2 of this note, we describe this situation. Briefly, there is an exact analogy between the question of approximation by finite rank linear mappings on compact sets and the question of approximation by finite rank holomorphic mappings on compact sets.

In §3, we study the theory of compact holomorphic mappings between Banach spaces. The results of this section can be applied to characterize when $H(E; C)$ has the approximation property, where $H(E; C)$ is endowed with topologies other than the compact-open. This is of interest because, for many purposes, the compact-open topology is not the natural topology on $H(E; C)$ (see for example [7]). In this note we are particularly concerned with the Nachbin "ported" topology τ_ω on $H(E; C)$. In §4, we characterize when $H(E; C)$, endowed with τ_ω, has the approximation property. This result relates compact holomorphic mappings to the approximation property for $(H(E; C), \tau_\omega)$ in a similar manner to the way compact linear mappings are related to the approximation property for (E', β).

We shall follow [6] for notation and terminology for holomorphic mappings on Banach spaces and [5] for notation and terminology for the approximation property.

2. E and F will denote complex Banach spaces. $H(E; F)$ denotes the space of holomorphic F-valued mappings on E; that is, mappings

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