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ON SPACES OF RIEMANN SURFACES WITH NODES[1]

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This is a summary of results, to be published in full elsewhere, which strengthen and refine the statements made in a previous announcement [1].

A compact Riemann surface with nodes of (arithmetic) genus $p > 1$ is a connected complex space S, on which there are $k = k(S) \geqslant 0$ points $P_1, \cdots, P_k$, called *nodes*, such that (i) every node P_j has a neighborhood isomorphic to the analytic set $\{z_1 z_2 = 0, |z_1| < 1, |z_2| < 1\}$, with P_j corresponding to $(0, 0)$; (ii) the set $S \backslash \{P_1, \cdots, P_k\}$ has $r \geqslant 1$ components $\Sigma_1, \cdots, \Sigma_r$, called *parts* of S, each Σ_i is a Riemann surface of some genus p_i, compact except for n_i punctures, with $3p_i - 3 + n_i \geqslant 0$, and $n_1 + \cdots + n_r = 2k$; and (iii) we have

$$p = (p_1 - 1) + \cdots + (p_r - 1) + k + 1.$$

Condition (ii) implies that every part carries a *Poincaré metric*, and condition (iii) is equivalent to the requirement that the total Poincaré area of S be $4\pi(p - 1)$.

From now on p is kept fixed and the letter S, with or without subscripts or superscripts, always denotes a surface with properties (i)–(iii). If $k(S) = 0$, S is called nonsingular; if $k(S) = 3p - 3$, S is called *terminal.*

A continuous surjection $f\colon S' \longrightarrow S$ is called a *deformation* if for every node $P \in S$, $f^{-1}(P)$ is either a node or a Jordan curve avoiding all nodes and, for every part Σ of S, $f^{-1}|\Sigma$ is an orientation preserving homeomorphism. Two deformations, $f\colon S' \longrightarrow S$ and $g\colon S'' \longrightarrow S$ are called *equivalent* if there are homeomorphisms $\varphi\colon S' \longrightarrow S''$ and $\psi\colon S \longrightarrow S$, homotopic to an isomorphism and to the identity, respectively, such that $g \circ \varphi = \psi \circ f$. The *deformation space* $D(S)$ consists of all equivalence classes $[f]$ of deformations onto S. To every node $P \in S$ belongs a *distinguished subset* consisting

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