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REGULARIZATION AND APPROXIMATION OF LINEAR OPERATOR EQUATIONS IN REPRODUCING KERNEL SPACES[1]

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1. **Introduction**. Let X and Y be real Hilbert spaces and let A be a linear operator with domain $\mathcal{D}(A) \subset X$ and range in Y. An element $u \in \mathcal{D}(A)$ is said to be a *least-squares* solution of the equation

$$Ax = y \tag{1}$$

for a given $y \in Y$ if $\inf\{\|Ax - y\| : x \in \mathcal{D}(A)\} = \|Au - y\|$. A *pseudosolution* of (1) for a given $y \in Y$ is a least-squares solution of minimal norm. Equation (1) *is well-posed* relative to the spaces X, Y if for each $y \in Y$, (1) has a unique pseudosolution which depends continuously on y; otherwise the equation is said to be *ill-posed.*

One objective of this research is to show, when X and Y are L_2-spaces of square-integrable functions, that the topology of reproducing kernel Hilbert spaces (RKHS) is an appropriate topology for the regularization of ill-posed linear operator equations, and to initiate a study of generalized inverses of linear operators acting between two RKHS. A second objective is to provide an approach to optimal approximations of linear operator equations in the context of RKHS, and to demonstrate the relation between the regularization operator of the equation $Af = g$ and the generalized inverse of A in an appropriate RKHS. (For some background on regularization methods see [3], [5], [9]; for generalized inverses see, for example, [4].)

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