SOLVABILITY ON MANIFOLDS BY QUADRATURES PERMITTING ONLY INTEGRALS

BY KUO-TSAI CHEN[1]

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Let M be a connected C^∞ manifold, and let $\rho\colon \widetilde{M} \to M$ be the universal covering map. Choose a base point $\widetilde{x}_0 \in \widetilde{M}$, and write $x_0 = \rho\widetilde{x}_0$. The fundamental group $\pi_1(M)$ is assumed to be finitely generated.

Let A be a subcomplex of the de Rham complex $\Lambda(M)$ satisfying the conditions:

(a) The subcomplex A is closed under the exterior product.

(b) The inclusion $A \subset \Lambda(M)$ induces an isomorphism $H(A) \approx H(\Lambda(M))$. Write $F_0 = \rho^*A^0$ and $\Omega = \rho^*A^1$. If $w \in A^1$ is a closed 1-form on M, then the integral $\int_{\widetilde{x}_0} \rho^*w$ is a function on $\widetilde{M}$ and can be regarded as a multivalued function on M. All such integrals together with 1 span a vector space F_1 of functions on $\widetilde{M}$ such that $F_0 \subset F_1$. For $r \geqslant 1$, define F_{r+1} to be the vector space of functions spanned by F_r and all $\int_{\widetilde{x}_0} w$, w being closed 1-forms belonging to the subspace $F_r\Omega$ of $\Lambda^1(\widetilde{M})$. It turns out that $\widetilde{F} = \bigcup_{r \geqslant 0} F_r$ is an algebra of functions on $\widetilde{M}$.

Recall that the lower central series of a group G consists of commutator subgroups G_r, $r \geqslant 1$, defined by $G_1 = G$ and $G_{r+1} = [G_r, G]$, $r \geqslant 1$. The lower central series is said to stabilize modulo torsion if G_r/G_{r+1} is finite for r sufficiently large. A group G is said to be torsion free residually nilpotent if each quotient G_r/G_{r+1} is torsion free and if $\bigcap G_r = \{e\}$.

The purpose of this note is to announce the next results, which will be proved in detail elsewhere.

THEOREM 1. *The algebra $\widetilde{F}$ is finitely generated over F if and only*