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THE SPACE OF CLASS α BAIRE FUNCTIONS

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ABSTRACT. Let X, Y be compact Hausdorff spaces and $B_\alpha^*(X)$, $B_\beta^*(Y)$, $0 \leqslant \alpha, \beta \leqslant \Omega$ (the first uncountable ordinal), the associated Banach spaces of bounded real-valued Baire functions of classes α and β. If $B_\alpha^*(X) \neq B_\beta^*(X)$ (which is the case if $\alpha \neq \beta$ and X is not dispersed), then $B_\alpha^*(X)$ is neither linearly isometric to $B_\beta^*(Y)$ nor equivalent to $B_\beta^*(Y)$ in several other ways. $B_\Omega^*(X)$ is linearly isometric to $B_\Omega^*(Y)$ if and only if X is Baire isomorphic to Y. For $1 \leqslant \alpha < \Omega$ the maximal ideal space of $B_\alpha^*(X)$ for a nondispersed compact space X is not an F-space.

1. Let X be a compact (more generally, completely regular) Hausdorff space and $C(X)$ the space of continuous real-valued functions on X. Let $B_0(X) = C(X)$, and inductively define $B_\alpha(X)$ for each ordinal $\alpha \leqslant \Omega$ (Ω denotes the first uncountable ordinal) to be the space of pointwise limits of sequences of functions in $\bigcup_{\xi<\alpha} B_\xi(X)$. Let $B_\alpha^*(X)$ be the space of bounded functions contained in $B_\alpha(X)$. With the pointwise operations $B_\alpha(X)$ and $B_\alpha^*(X)$ are lattice-ordered algebras. With the supremum norm $B_\alpha^*(X)$ is a Banach algebra (see [4, §41]).

The Baire sets of X of multiplicative class α, denoted by $Z_\alpha(X)$, are defined to be the zero sets of functions in $B_\alpha^*(X)$. Those of additive class α, denoted by $CZ_\alpha(X)$, are defined as the complements of sets in $Z_\alpha(X)$. Finally, those of ambiguous class α, denoted by $A_\alpha(X)$, are the sets which are simultaneously in $Z_\alpha(X)$ and $CZ_\alpha(X)$. With the set-theoretic operations of union and intersection, $A_\alpha(X)$ is a Boolean algebra for each $\alpha \leqslant \Omega$. The sets of exactly ambiguous class α, denoted by $EA_\alpha(X)$, are those in $A_\alpha(X)\backslash\bigcup_{\xi<\alpha} A_\xi(X)$. The sets of exactly additive and exactly multiplicative class α are defined analogously. The class of all Baire subsets of X is $Z_\Omega(X)$.

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