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SUBCOMPLEXES OF POINCARÉ COMPLEXES

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1. **Introduction.** This paper is concerned with the following situation. Suppose we are given a simplicial complex X which is a Poincaré duality space for some (twisted) coefficients, and that K is a subcomplex of X which is itself a Poincaré duality space; when can we deduce that the inclusion of K in X is homotopic to a Poincaré embedding?

There are two interesting cases of the above situation, the first is given by a theorem of Bredon [1], which states that if Z_p acts on a simplicial complex which is a Z_p-Poincaré duality space, then each component of the fixed point set is a Z_p-Poincaré duality space. The other case is that of the product of a Poincaré duality space with itself, triangulated so that the diagonal is a subcomplex. We shall give a condition sufficient to obtain an embedding in these two cases, subject to certain dimensional restrictions.

2. **Normally embedded subcomplexes.** For the purpose of simplicity of exposition, we shall restrict ourselves to the simply-connected case. We shall begin by discussing the case of integer coefficients, and will describe the modifications required for the Z_p case when we discuss the Bredon theorem. We assume familiarity with the basic definitions of Poincaré complex, Poincaré embedding (they can be found for instance in Levitt [3]), and with the concept of a normal space and normal pair as defined by Quinn [4].

It is known [4] that the mapping cylinder of a normal map is a normal pair. Furthermore a Poincaré complex is a normal space with the spherical fibration given by the Spivak normal bundle construction.

A Poincaré complex of formal dimension n, that is presented as an n-dimensional simplicial complex, will be called a triangulated Poincaré complex.

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