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OBSTRUCTIONS TO TRANSVERSALITY FOR COMPACT LIE GROUPS

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Throughout G is a compact Lie group which is topologically cyclic with dense generator g. Let N and M be smooth G manifolds without boundary and $Y \subset M$ a closed invariant submanifold. All manifolds are oriented and G preserves orientation. Let $f: N \to M$ be a proper G map. When is f properly G homotopic to a map γ which is transverse regular to $Y \subset M$, written $\gamma \pitchfork Y$? We introduce obstructions which show that transversality is a global phenomena in contrast to the case $G=1$ where everything is local and trivial.

Without loss of generality, we may assume that $f^g: N^g \to M^g$ is transverse to Y^g and set $X^g = (f^g)^{-1}(Y^g)$. For each oriented real G vector bundle v over X^g such that the G representation on each fiber of v has no trivial factor and g preserves orientation on each fiber, let $\lambda_\pm(v)$ be the $\pm$ eigenbundles of the canonical involution τ on $\lambda(v \otimes C) = \sum \lambda^i(v \otimes C)$ constructed from the orientation and an inner product on v. Let $\lambda_{-1}(v \otimes C) = \sum (-1)^i \lambda^i(v \otimes C)$, $I^{X^g} \in K_G(TX^g)$ be the index class of X^g, i.e. the symbol of the operator D^+. See [**1**, p. 575]. Let $\mathscr{P} \subset R(G)$ be the prime ideal of characters $\{X \in R(G) \mid X(g) = 0\}$ and

$$\text{(i)} \qquad \mathscr{B}(v) = \frac{\lambda_+(v) - \lambda_-(v)}{\lambda_{-1}(v \otimes C)} \cdot I^{X^g} \in K_G(TX^g)_{\mathscr{P}}.$$

Let $f: X \to Y$ be a G map. If f is an embedding there is a homomorphism $f!: K_G(TX) \to K_G(TY)$ [**1**]. By taking the product of Y with a real G module and using the Thom isomorphism for complex G vector bundles, we may assume that $f!$ is defined for any map f and denote it by f_*. The normal bundle of Y in M is denoted by $\nu(Y, M)$. Its restriction to Y^g has a splitting

$$\text{(ii)} \qquad (i^g)^* \nu(Y, M) = \nu(Y, M)^g + \nu_2(Y, M),$$

where $\nu(Y, M)^g$ is the subbundle of points fixed by g and $i^g: Y^g \to Y$ is

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