BULLETIN OF THE
AMERICAN MATHEMATICAL SOCIETY
Volume 80, Number 6, November 1974

VARIETIES OF SMALL CODIMENSION IN PROJECTIVE SPACE[1]

BY ROBIN HARTSHORNE

Introduction. I would like to begin by stating a conjecture. While I am not convinced of the truth of this statement, I think it is useful to crystallize one's ideas, and to have a particular problem in mind. Then for the remainder of the talk, I propose to examine this question in a rather general way from a number of different perspectives. This will give me an opportunity to report on recent work in several areas of algebraic geometry, and at the same time to mention a number of open problems.

Let k be an algebraically closed field. Let $\boldsymbol{P}^n$ be the n-dimensional projective space over k. Let $Y \subseteq \boldsymbol{P}^n$ be a nonsingular subvariety of dimension r. We say that Y is a *complete intersection* in $\boldsymbol{P}^n$ if one can find $n-r$ hypersurfaces $H_1, \cdots, H_{n-r}$, such that $Y=H_1 \cap \cdots \cap H_{n-r}$, and such that this intersection is *transversal*, i.e. the hypersurfaces H_i are nonsingular at all points of Y, and their tangent hyperplanes intersect properly at each point of Y. In algebraic terms, Y is a complete intersection if and only if its homogeneous prime ideal $I(Y) \subseteq k[x_0, \cdots, x_n]$ can be generated by exactly $n-r$ homogeneous polynomials.

Conjecture. If Y is a nonsingular subvariety of dimension r of $\boldsymbol{P}^n$, and if $r > \frac{2}{3}n$, then Y is a complete intersection.

The paper is divided into six sections:

§1. Representing cohomology classes by subvarieties.
§2. Cohomological properties of the subvariety.
§3. Examples. Subvarieties of small degree.
§4. Embedding varieties in projective space.
§5. Connections with local algebra.
§6. Existence of vector bundles on $\boldsymbol{P}^n$.

1. **Representing cohomology classes by subvarieties.** To give perspective on our conjecture, let us consider some more general questions. Let X be a nonsingular algebraic variety, and let Y be a (possibly singular) subvariety. Then we can consider the cohomology class of Y, in any

An expanded version of an invited address delivered before the Missoula, Montana Meeting of the Society on August 24, 1973; received by the editors February 5, 1974.

AMS (MOS) subject classifications (1970). Primary 14C30, 14M10; Secondary 13H10.

Key words and phrases. Complete intersection, vector bundles, Gorenstein rings.

[1] Research supported by an NSF grant.