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GENERALIZED GRADIENT FIELDS AND ELECTRICAL CIRCUITS[1]

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1. **Introduction.** In *On the mathematical foundations of electrical circuit theory*, Smale [**S.1**] proposes the following two problems.

Problem 1.1. What can one say about the dynamical systems which are gradient systems with respect to a nondegenerate indefinite metric, say on a compact manifold.

Problem 1.2. Can one always regularize the equations of (1.6) [**S.1**], by adding arbitrarily small inductors and capacitors to the circuit appropriately? How? By regularizing we mean obtaining new equations which have the property $\pi:\sum\to\mathscr{L}\times\mathscr{C}'$ is a local diffeomorphism.

Furthermore he makes the following conjecture

CONJECTURE 1.3. Suppose $X=\text{grad}(\omega)$ is the gradient of a closed 1-form with respect to a Riemannian metric on a compact manifold M. Suppose further that ω is not cohomologous to zero and that X is well behaved in the sense that it satisfies the conditions of [**S.2**, (2.2)]. Then X has a closed orbit, not a point, which is aymptotically stable (i.e. a sink).

In this work we give a counterexample to this conjecture. Furthermore we reformulate it, solving the new version in the case $\dim M=2$. For Problem 1.1 we obtain generic properties for the generalized gradient fields as in the Kupka-Smale theorem. Moreover we characterize structural stability for these types of vector fields in the case M is compact, orientable, and $\dim=2$. For Problem 1.2 we give a counterexample in the general case and solve the problem imposing conditions on the resistors of the circuit.

Before we state the theorems we need some definitions and notations. M will be a C^∞ manifold (with or without boundary), $TM\oplus TM=\{(p, v, w)|p\in M, v, w\in TM_p\}$.

DEFINITION 1.4. A metric C^r on M is a C^r map $\mu: TM\oplus TM\to \boldsymbol{R}$, such that for each $p\in M$, the map $\mu_p: TM_p\times TM_p\to\boldsymbol{R}$ given by $\mu_p(v, w)=\mu(p, v, w)$ is bilinear symmetric. We say that μ is a nondegenerate metric on M if for each $p\in M$, μ_p is a nondegenerate bilinear form on TM_p.

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[1] This is a resume of my dissertation, developed at IMPA under the guidance of Professor Cesar Camacho.