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RADICAL EMBEDDING, GENUS, AND TOROIDAL DERIVATIONS OF NILPOTENT ASSOCIATIVE ALGEBRAS

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ABSTRACT. The author continues to discuss this problem: given a nonzero nilpotent finite-dimensional associative algebra N over the perfect field k, describe the set of unital associative k-algebras A satisfying the equation $\operatorname{rad} A = N$, together with the "nowhere triviality" condition $\operatorname{Ann}_A N \subset N$. In this paper the Lie homomorphism $\delta: S_{\mathrm{Lie}} \to \operatorname{Der}_k N$ induced by bracketing (where A has Wedderburn decomposition as semidirect sum $S+N$) is studied as follows: (i) the kernel and image of δ are computed; (ii) conditioning the derivation algebra $\operatorname{Der}_k N$ conditions the semisimple S; (iii) for instance, $\operatorname{Der}_k N$ solvable implies that S is a direct sum of fields; (iv) those tori in $\operatorname{Der}_k N$ of the form δS are characterized in terms of their 0-weightspace in N.

1. **Introduction.** For previous discussions, see Hall [**2**] and Flanigan [**1**]. Throughout, N is a given finite-dimensional nilpotent k-algebra with k perfect. We seek those semisimple k-algebras S which satisfy the following conditions.

(1.1) DEFINITION [**1**]. *N accepts S as a nowhere trivial Wedderburn factor* if there is a unital associative k-algebra S A such that (i) $A \simeq N+S$ (Wedderburn decomposition), and (ii) $S \cap \operatorname{Ann}_A N = (0)$.

Note that (ii) forces A to be finite dimensional, and that $N \neq (0)$ implies $S \neq (0)$. In [**1**] we examined candidates S for acceptance by considering such invariants of N as its quotients N/N^i and its graded form $\operatorname{gr} N$. Now we utilize the Lie algebra $\operatorname{Der}_k N$ of k-algebra derivations $N \to N$ by noting that, if N accepts S as in (1.1), then there is a Lie homomorphism

$$\delta: S_{\mathrm{Lie}} \to \operatorname{Der}_k N \tag{1.2}$$

with $\delta(b)x = [b, x] = bx - xb$ for all x in N, b in S, and with the products taken in A.

We are particularly interested in those S which are direct sums of fields. *Reason*: the center of *every* semisimple algebra accepted by N would be of this type. These direct sums of fields are determined by the

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