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A CONVERGENT FAMILY OF DIFFUSION PROCESSES WHOSE DIFFUSION COEFFICENTS DIVERGE

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1. **Introduction.** The problem of deriving suitable hypotheses from which one can conclude the weak convergence of a family of diffusion processes $x_n(t, \omega)$ to a limiting diffusion process $x(t, \omega)$ as $n \to \infty$ has attracted much attention in recent years. One popular approach is to study the asymptotic behavior of the diffusion coefficients. Specifically let us assume that $\{x_n(t, \omega), 1 \leqq n < +\infty\}$ and $x(t, \omega)$ are one-dimensional, strong Markov processes with continuous paths and stationary transition probabilities. Assume further that the infinitesimal generators G_n and G of the corresponding semigroups

$$T_n(t)f(x) = E_x f(x_n(t, \omega)) \quad \text{and} \quad T(t)f(x) = E_x f(x(t, \omega))$$

are classical second order differential operators of the form:

$$(1) \qquad \begin{aligned} G_n f(x) &= a_n(x)f''(x) + b_n(x)f'(x), \qquad a_n(x) > 0, \\ Gf(x) &= a(x)f''(x) + b(x)f'(x), \qquad a(x) > 0. \end{aligned}$$

Under sufficiently stringent hypotheses, Skorohod [**8**], Borovkov [**1**], Stroock-Varadhan [**9**], among others, have shown that a condition of the form

$$(2) \qquad \lim_{n \to \infty} a_n(x) = a(x), \qquad \lim_{n \to \infty} b_n(x) = b(x)$$

is sufficient to conclude convergence of the semigroups, i.e.

$$(3) \qquad \lim_{n \to \infty} T_n(t)f(x) = T(\ )f(x)$$

for all f in a sufficiently large class of functions. It is known however that the infinitesimal generator G of the diffusion process $x(t, \omega)$ need not be a classical second order differential operator but instead can be one

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