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FINITE-DIMENSIONAL REPRESENTATIONS OF SEPARABLE C^*-ALGEBRAS

BY CARL PEARCY AND NORBERTO SALINAS

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Let $\mathscr{H}$ be a separable, infinite-dimensional, complex Hilbert space, and let $\mathscr{L}(\mathscr{H})$ denote the algebra of all bounded linear operators on $\mathscr{H}$. Furthermore, let $\mathscr{K}$ denote the (norm-closed) ideal of all compact operators in $\mathscr{L}(\mathscr{H})$, and let $\pi:\mathscr{L}(\mathscr{H})\rightarrow\mathscr{L}(\mathscr{H})/\mathscr{K}$ denote the canonical quotient map of $\mathscr{L}(\mathscr{H})$ onto the Calkin algebra. If T is any operator in $\mathscr{L}(\mathscr{H})$, we shall denote by $\mathscr{C}^*(T)$ the C^*-algebra generated by T and $1_{\mathscr{H}}$. Moreover, the C^*-algebra $\pi(\mathscr{C}^*(T))$, which is clearly the C^*-subalgebra of the Calkin algebra generated by $\pi(T)$ and 1, will be denoted by $\mathscr{C}_e^*(T)$. If $\mathscr{A}$ is any C^*-algebra, an *n-dimensional representation* of $\mathscr{A}$ is, by definition, a $*$-algebra homomorphism φ of $\mathscr{A}$ into the C^*-algebra $\boldsymbol{M}_n$ of all $n\times n$ complex matrices such that $\varphi(1)=1$. Such a representation φ will be called *irreducible* if $\varphi(\mathscr{A})=\boldsymbol{M}_n$.

The first objective of this note is to announce the following theorem, which gives, via the standard decomposition theory, a characterization of all finite-dimensional representations of a separable C^*-algebra. See [2].

THEOREM 1. *Let $\mathscr{A}$ be a separable C^*-subalgebra of $\mathscr{L}(\mathscr{H})$, and let φ be an irreducible n-dimensional representation of $\mathscr{A}$. Then, either*

(a) $\mathscr{A}\cap\mathscr{K}\subset$ kernel φ *(equivalently, there exists an n-dimensional representation $\tilde{\varphi}$ of the C^*-algebra $\pi(\mathscr{A})$ such that $\varphi(A)=\tilde{\varphi}(\pi(A))$ for every A in $\mathscr{A}$), in which case there exists a projection P in $\mathscr{L}(\mathscr{H})$ with infinite rank and nullity such that $\pi(P)$ commutes with the algebra $\pi(\mathscr{A})$, and there exists a $*$-algebra isomorphism ψ from the C^*-algebra $\pi(\mathscr{A})\pi(P)$ $(=\{\pi(A)\pi(P): A\in\mathscr{A}\})$ onto $\boldsymbol{M}_n$ such that $\varphi(A)=\psi(\pi(A)\pi(P))$ for every A in $\mathscr{A}$, or*

(b) $\mathscr{A}\cap\mathscr{K}\not\subset$ kernel φ, *in which case there exist a projection Q in $\mathscr{A}$ of finite rank that commutes with $\mathscr{A}$ and a $*$-algebra isomorphism η from the C^*-algebra $\mathscr{A}Q$ $(=\{AQ:A\in\mathscr{A}\})$ onto $\boldsymbol{M}_n$ such that $\varphi(A)=\eta(AQ)$ for every A in $\mathscr{A}$.*

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