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DIFFERENTIABLE Z_p ACTIONS ON HOMOTOPY SPHERES[1]

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The results of [4] proved that many exotic spheres do not admit smooth actions of relatively high-dimensional compact Lie groups (all group actions considered in this paper are assumed to be effective). It was clear that stronger results should hold in certain cases, and this was confirmed in [5]. A notable feature of [5] is the use of nonexistence theorems for certain smooth circle actions to prove nonexistence theorems for large Lie group actions. These theorems for circle actions actually reflect much stronger nonexistence results for smooth Z_p actions, the proofs of which are outlined in this paper.

Recently H. B. Lawson and S. T. Yau obtained other (frequently much stronger) nonexistence theorems for connected *nonabelian* actions on exotic spheres using differential-geometric methods and results of N. Hitchin. Since our methods and [2] readily yield nonexistence theorems for toral actions, in some sense the topological methods of this paper are complementary to these geometric methods.

As an illustration of the sort of results obtainable by our methods, we give examples of exotic spheres with no effective smooth torus actions.

1. **Normal invariants of homology equivalences.** A *homotopy smoothing* of a smooth manifold M is a pair (X, f) such that $f: X \to M$ is a homotopy equivalence; a fundamental construction assigns to each homotopy smoothing a normal invariant $\eta(X, f) \in [M, F/O]$. In fact, normal invariants are definable for pairs (X, f) where f is merely a homology equivalence with respect to a subring of the rationals.[3] All such subrings have the form Z_l, the integers with inverses of all primes *not* in l adjoined, and the generalized normal invariant takes its values in $[M, F/O_l] \simeq [M, F/O]_l$ (see [8, Chapter II] for the relevant localization theory). Many formal properties of ordinary normal invariants which are useful in calculation

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[3] Various generalizations of normal invariants to homology equivalences have been previously studied by L. Jones, S. Cappell and J. Shaneson, and W. Browder (and probably others).