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ORTHOGONALITY AND NONLINEAR FUNCTIONALS

BY S. GUDDER AND D. STRAWTHER

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Let X be a vector space of functions. In practice, X is taken to be (i) the set of continuous functions on some type of topological space or (ii) a set of measurable functions on a measure space. We say that $x, y \in X$ are *orthogonal in the lattice theoretic sense* ($x \perp_L y$) if $\{t:x(t)y(t)\neq 0\}$ is $\varnothing$ in case (i) or of measure zero in case (ii). A real valued functional $f:X\to R$ is *L-additive* if $f(x+y)=f(x)+f(y)$ whenever $x \perp_L y$. If f is L-additive and satisfies certain continuity or boundedness conditions then f admits an integral representation giving a nonlinear generalization of the Riesz theorem. Such representations have been obtained for case (i) in **[1]**, **[4]** and for case (ii) in **[2]**, **[3]**, **[10]**, **[11]**, **[12]**, **[14]**. Although the above orthogonality is important for certain applications **[5]**, **[9]**, **[13]**, in this note we consider orthogonalities which are standard in inner product and normed spaces. To some extent our results are less general than those in the above papers since the standard orthogonality is weaker than lattice theoretic orthogonality. On the other hand, some of our theorems apply to more general vector spaces than the above and furthermore we have obtained results for a more general class of functionals which we call orthogonally monotone functionals. Finally, we use our results to solve a nonlinear functional equation and give an application for the solution.

An *orthogonality vector space* is a real vector space X with $\dim X \geqq 2$ on which there is defined a relation $x\perp y$ such that

(01) $0\perp x$, $x\perp 0$ for all $x\in X$,

(02) if $x\perp y$ and $x, y\neq 0$ then x, y are linearly independent,

(03) if $x\perp y$ then $\alpha x\perp \beta y$ for all $\alpha, \beta \in R$,

(04) if B is a two-dimensional subspace of X, then for every $0\neq x\in B$, there exists $0\neq y\in B$ such that $x\perp y$ and $x+y\perp x-y$.

It is easily seen that any real vector space of dimension $\geqq 2$ is an orthogonality vector space if we define $0\perp x$, $x\perp 0$ for all x and if $x, y\neq 0$ then $x\perp y$ iff x, y are linearly independent. It is also clear that an inner product space is an orthogonality vector space under the standard orthogonality

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