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TORSION ALGEBRAIC CYCLES, K_2, AND BRAUER GROUPS OF FUNCTION FIELDS

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0. **Introduction.** Let F be a field, and let $H^*(F, \mu_n)$ denote the Galois cohomology of $\mathrm{Gal}(F_s/F)$ with coefficients in the group μ_n of nth roots of 1 for some fixed n prime to char F. Bass and Tate have shown that the natural pairing

$$F^*/F^{*n} \times F^*/F^{*n} = H^1(F, \mu_n) \times H^1(F, \mu_n) \xrightarrow{\text{cup product}} H^2(F, \mu_n^{\otimes 2})$$

is a symbol on F. In other words there is an induced homomorphism (nth power norm residue symbol) of the Milnor K_2 group [7], $R_{n,F}: K_2(F)/nK_2(F) \to H^2(F, \mu_n^{\otimes 2})$.

Tate showed that $R_{n,F}$ is an isomorphism where F is a global field, and asked whether an analogous result held for arbitrary fields. The situation is particularly interesting when $\mu_n \subset F$, because in this case

$$H^2(F, \mu_n^{\otimes 2}) \cong H^2(F, \mu_n) \otimes \mu_n \cong_n \mathrm{Br}(F) \otimes \mu_n$$

($\mathrm{Br}(F)$=Brauer group of F). Surjectivity of R_n implies, for example, that every division algebra with exponent n and center F is split by an *abelian* extension field of F. The question of surjectivity for R_2, for example, amounts to the classical question of whether a division algebra of exponent 2 is equivalent to a tensor product of quaternion algebras.

In this note I will consider the case F=function field of an algebraic variety X over a ground field k. I will give some partial results of an algebraic nature, and sketch some relations between Tate's question for F, and the global geometry of X. Detailed proofs are available in preprint form.

1. Algebraic results.

THEOREM (1.1). *Let $F=k(t_1, \cdots, t_r)$ be a rational function field in r variables over a field k, and let n be an integer prime to* char k. *Then the*

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