BULLETIN OF THE
AMERICAN MATHEMATICAL SOCIETY
Volume 80, Number 5, September 1974

AN EMBEDDING-OBSTRUCTION FOR PROJECTIVE VARIETIES[1]

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Communicated by A. Mattuck, January 29, 1974

A classical problem in differential topology is the following: Let X be a compact n-dimensional differentiable manifold (without boundary). Then compute the least integer $m=m(X)$ such that X may be embedded into $\boldsymbol{R}^m$. Usually this question is attacked as follows (see Atiyah [**1**]): (a) An upper bound for m is obtained by exhibiting explicit embeddings, and (b) a lower bound is obtained by certain homotopy invariants.

The forthcoming paper [**2**] deals with an algebro-geometric counterpart to the problem mentioned above: Let X be a nonsingular, projective k-variety embedded in some projective space $\boldsymbol{P}_k^N$ by the embedding i. For simplicity we assume the field k to be algebraically closed, but the results of [**2**] still hold under the weaker assumption that k is infinite. The main result is that the least integer $m=m(X, i)$, such that X can be embedded into $\boldsymbol{P}_k^m$ via a projection from $\boldsymbol{P}_k^N$, is *effectively computed in terms of the degrees of the Chern-classes of X.*

More precisely, let $X \subsetneq \boldsymbol{P}_k^N$ be an n-dimensional nonsingular projective variety, embedded in $\boldsymbol{P}_k^N$. Let $c_i=c_i(X)=c_i(\Omega^1_{X/k}) \in A(X)$ be the Chern-classes of X, where $A(X)$ denotes the Chow-ring of X. Consider the formal inverse of the alternating Chern-polynomial:

$$\left[\sum_{i=0}^{n}(-1)^i c_i T^i\right]^{-1} = \sum_{i=0}^{\infty} f_i T^i.$$

Here $f_i=0$ for $i>n$. Let $d_i=\deg(f_i)$ with respect to the embedding $i: X \subsetneq \boldsymbol{P}_k^N$. In particular $d_0=\deg(i(X))=d$. Define

$$B_X(T) = \left(\sum_{i=0}^{n} d_i T^i\right)\left(\sum_{i=0}^{2n+1}\binom{2n+2}{i}T^i\right) = B_0 + B_1 T + \cdots,$$

AMS (MOS) subject classifications (1970). Primary 14E25.

Key words and phrases. Embedding, obstruction, Chern-classes.

[1] Research supported in part by NSF under grant GP29026X, and by the Norwegian Research Council for Science and the Humanities.