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SOME THEOREMS ON C-FUNCTIONS

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The purpose of this note is to announce certain results I have obtained about the behavior of the Harish-Chandra C-function as a meromorphic function. The notation and terminology, if not explained, are that of [2], [3], or [6].

1. **The C-ring.** Let (P, A) be a fixed parabolic pair of a semisimple Lie group G having finite center, $P=MAN$ the corresponding Langlands decomposition, K a fixed maximal compact subgroup. Let $\mathcal{G}$, $\mathcal{K}$, $\mathcal{K}_M$, $\mathcal{M}$ be the universal enveloping algebras of G, K, K_M, and M, respectively ($K_M=K\cap M$)—i.e. of their complexified Lie algebras $\mathfrak{g}_C$, $\mathfrak{k}_C$, $\mathfrak{k}_{M,C}$, $\mathfrak{m}_C$. Let $b\to b^\iota$ ($b \in \mathcal{G}$) denote the unique anti-automorphism of $\mathcal{G}$ such that $X^\iota=-X$ ($X\in \mathfrak{g}$). Consider $\mathcal{K}$ to be a right $\mathcal{K}_M$-module via the multiplication in $\mathcal{K}: b\circ d=bd$ ($b\in\mathcal{K}$, $d\in\mathcal{K}_M$), and consider $\mathcal{M}$ to be a left $\mathcal{K}_M$-module via the operation $d\circ c=cd^\iota$ ($d\in\mathcal{K}_M$, $c\in\mathcal{M}$). We can then form the tensor product $\mathcal{K}\otimes_{\mathcal{K}_M}\mathcal{M}$ of $\mathcal{K}_M$-modules. (We write $b\hat{\otimes} c$ for elements of $\mathcal{K}\otimes_{\mathcal{K}_M}\mathcal{M}$, $b\otimes c$ for elements of $\mathcal{K}\otimes\mathcal{M}$.) The group K_M acts on $\mathcal{K}\otimes_{\mathcal{K}_M}\mathcal{M}$ via the (well-defined) representation $\rho: \rho(m)(b\hat{\otimes} c)=b^m\hat{\otimes} c^m$ ($b\in\mathcal{K}$, $c\in\mathcal{M}$, $m\in K_M$). Let $(\mathcal{K}\otimes_{\mathcal{K}_M}\mathcal{M})^{K_M}$ denote the K_M-invariants.

PROPOSITION 1. *$(\mathcal{K}\otimes_{\mathcal{K}_M}\mathcal{M})^{K_M}$ is a ring (i.e., the "obvious" multiplication is well defined). In fact, it is a left and right Noetherian integral domain (noncommutative, in general), hence has a quotient division algebra.*

We refer to $(\mathcal{K}\otimes_{\mathcal{K}_M}\mathcal{M})^{K_M}$ as the C-ring associated to the pair (P, A).

Let τ be a left or double representation of K on a finite-dimensional Hilbert space V. Then there exists a representation λ_τ of the ring $\mathcal{K}\otimes\mathcal{M}$ on $C^\infty(M:V)$ defined as follows:

$$\lambda_\tau(b\otimes c)\psi(m)=\tau(b)\psi(c_l^\iota m)\qquad (b\in\mathcal{K}, c\in\mathcal{M}, m\in M, \psi\in C^\infty(M:V)).$$

Let $C^\infty(M, \tau_M)$ denote the space of $\psi\in C^\infty(M:V)$ such that

$$\tau(k)\psi(m)=\psi(km)\qquad (k\in K_M, m\in M)$$

if τ is a left representation of K or such that

$$\tau(k_1)\psi(m)\tau(k_2)=\psi(k_1mk_2)\qquad (k_1, k_2\in K_M, m\in M)$$

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